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Table of contents

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Developing a mathematical model of instrumental examination of patients

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ABSTRACT: The processing steps of information obtained as a result of generating digital biomedical signals and images with locally concentrated signs are formalized by an aggregate of sets of characteristics of the instrumental examination process. A mathematical model of the instrumental examination process of patients is developed. Critical states of the instrumental examination are identified with the model introduced.

1 INTRODUCTION

The introduction of computer and information technologies into medical practice has led to the creation of a variety of medical information systems (MIS), the main purpose of which is to improve the effectiveness of management processes (medical diagnostic, administrative-economic, financial and other activities) in health care to raise the quality of medical care for the population. The most common MIS were obtained as part of diagnostic complexes in the form of MIS for laboratory diagnostic studies, with which various instrumental examinations of patients are carried out. One of the varieties of MIS are biomedical decision support systems (DSS), as part of hardware and software diagnostic complexes, which support decision-making based on various models (for example, heuristic or mathematical). When creating such models, it is important to take into account the specifics of both the representation and the manifestations of clinical information.

Currently, there is a wide range of computer diagnostic systems in various subject domains of medicine (Povoroznyuk 2011, Kotyra 2014, Zlepko et al. 2016, Kovalenko et al. 2017, Wójcik & Smolarz 2017), in which various mathematical methods of decision support are used e.g. deterministic logic (Kobrinsky 2005), probabilistic approach (Sadegh-Zadeh 2011,

Selivanova et al. 2016), fuzzy logic (Innocent et al. 2005, Rotshtein et al. 2006, Rotshtein et al. 2008, Smolarz et al. 2012), neural networks (Timchenko et al. 2002, Ceylana et al. 2009), etc., and modern information technologies, including telemedicine (Krawczyk 3003, Yang et al. 2015, Vladismirsky 2011, Hwang 2016, Serkova et al. 2017). The results of instrumental examinations by medical specialists are the conclusions that are formed from the morphological analysis of biomedical signals and images (BMS/I) with locally concentrated features (LCF), so the methods of their processing for the purpose of determining the diagnostic features are given great attention (Faynzilberg 2015, Trzupek et al. 2011). However, at the present time there is no single formalised approach to solving the problem of morphological analysis of BMS/I with LCF for constructing biomedical DSS, and neither are there any stages of transformation of information obtained as a result of instrumental examination of patients. Thus, in order to improve the effectiveness of instrumental examination, it is necessary to perform a system analysis of the decision-making process, which is based on the processing of the BMS/I with the LCF, in order to identify the critical elements of the decision support system that can lead to the making of incorrect decisions or refusal to make decisions.

The aim of the research is to develop a mathematical model of instrumental examination of patients, with the help of which it would be possible to formalise the knowledge generated as a result of biomedical information processing, including the analysis of biomedical signals and images with locally concentrated signs, to improve the efficiency of providing medical services and minimising the risk of medical errors.

Objectives of the research:

1. To formalise the processing steps of information obtained from the analysis of biomedical signals and images with locally concentrated features, while conducting an instrumental examination of patients.
2. To develop a mathematical model of the process of instrumental examination of patients, taking into account the stages of information transformation.

2 SOLUTION

To construct a mathematical model of the instrumental examination process, let us formalise the knowledge generated as a result of processing digital BMS/I with LCF by an aggregate of sets of characteristics of the instrumental examination process (Burtsev et al. 2013). Since a one-dimensional signal can be considered as a particular case of a two-dimensional signal (image), then, in order to formalise the posed problem of the morphological analysis of BMS/I with LCF, the indices determining the dimensionality of the signal are omitted.

Based on the analysis of the stages of biomedical information processing during laboratory diagnostic studies, the following mathematical model of the instrumental examination process is proposed:

$$M_M = \langle MR, X, S, V, B, C, D, \tilde{D}, R, f \rangle, \quad (1)$$

where $MR = \{ mr_i | i \in \{1,2,\ldots,n_{MR}\} \}$ is a set of patient data from medical records; $X = \{ x_i[\cdot] | i \in \{1,2,\ldots,n_X\} \}$ is a set of registered digital BMS/I with LCF; $S = \{ s_i \in S^{(X)} \cup S^{(A)} \cup S^{(Prev)} | i \in \{1,2,\ldots,n_S\} \}$ is a set of diagnostic features; $S^{(X)}$ is a subset of diagnostic features of BMS/I with LCF; $S^{(A)}$ is a subset of diagnostic features of anamnesis; $S^{(Prev)}$ is a subset of features derived from previous examinations; $V = \{ V_j | j \in \{1,2,\ldots,n_S\} \}$ is a set of values of diagnostic features S; $B = \{ B_j | j \in \{1,2,\ldots,n_S\} \}$ is a set of value ranges of diagnostic features S (diagnostic ranges); $C = \{ C_i | i \in \{1,2,\ldots,n_C\} \}$ is a set of symptom-complexes; $D = \{ d_k | k \in \{1,2,\ldots,n_D\} \}$ is a set of possible diagnoses in a given subject domain, i.e. the alphabet of diagnoses; $\tilde{D} = \{ \tilde{D}_i | i \in \{1,2,\ldots,n_{\tilde{D}}\} \}$ is a set of fuzzy diagnostic conclusions; $R = \{ r_i | i \in \{1,2,\ldots,n_R\} \}$ is a set of recommendations; $f = \{ f_{XS}, f_{MrS}, f_{SV}, f_{VB}, f_{BC}, f_{CD}, f_{DR} \}$

is a set of correspondences between pairs of appropriate sets $MR, X, S, V, B, C, D, \tilde{D}, R; n_{MR}, n_X, n_S, n_C, n_D, n_{\tilde{D}}, n_R \in \mathbb{N}$ are the cardinalities of the corresponding sets $MR, X, S, C, D, \tilde{D}, R$; D is the natural number set.

Let us consider in detail the elements of the proposed mathematical model M_M of the instrumental examination process.

In the instrumental examination process there is a registered set of BMS/I with LCF provided by the survey protocol in this subject domain: $X = \left\{ x_i[\cdot] \middle| i \in \{1,2,\ldots,n_X\} \right\}$, where $x_i[\cdot]$ is the i-th BMS/I with LCF; n_X is the number of BMS/I with LCF. For example, the set of ECG leads is formed as a result of electrocardiographic examination, and the set of projections of the left and right mammary glands is formed as a result of mammography examination, and so on.

Also, in accordance with regulatory documents there is a determined subset of diagnostic features $S_i^{(X)} = \left\{ s_{ij}^{(X)} \middle| j \in \{1,2,\ldots,n_{X_i}\} \right\}$ by processing each BMS/I $x_i[\cdot]$, where $s_{ij}^{(X)}$ is the j-th feature of the i-th BMS/I with LCF; n_{X_i} is the number of diagnostic features for the i-th BMS/I, at that $S_i^{(X)} \cap S_j^{(X)} = \varnothing$ if $i \neq j$ and $\bigcup_{i=1}^{n_X} S_i^{(X)} = S^{(X)}$; $S^{(X)} \neq \varnothing$ is a set of diagnostic features for all BMS/I with LCF, i.e. for the set X, at that $n_{S^{(X)}} = \left| S^{(X)} \right| = \sum_{i=1}^{n_X} n_{X_i}$ is the total number of diagnostic features. In addition, a subset of diagnostic features of an anamnesis $S^{(A)} = \left\{ s_i^{(A)} \middle| i \in \{1,2,\ldots,n_{S^{(A)}}\} \right\}$ and a subset of features derived from previous examinations $S^{(Prev)} = \left\{ s_i^{(Prev)} \middle| i \in \{1,2,\ldots,n_{S^{(Prev)}}\} \right\}$ are formed from the patient's medical records, where $s_i^{(A)}, s_i^{(Prev)}$ are the i-th feature of anamnesis and previous examinations, respectively; $n_{S^{(A)}}, n_{S^{(Prev)}}$ are the number of diagnostic features of anamnesis and previous examinations, respectively. It must be noted that if the diagnostic features of anamnesis and/or previous examinations are not taken into account, then $S^{(Prev)} = \varnothing$ $(n_{S^{(A)}} = 0)$ and/or $S^{(A)} = \varnothing$ $(n_{S^{(Prev)}} = 0)$.

Then the set of all diagnostic features can be written as $S = \{ s_j \left| s_j \in S^{(X)} \cup S^{(A)} \cup S^{(Prev)}, \right. \forall j \in \{1,2,\ldots,n_S\} \}$, where S_j is the j-th diagnostic feature; $n_S = \left| S^{(X)} \right| + \left| S^{(A)} \right| + \left| S^{(Prev)} \right| = n_{S^{(X)}} + n_{S^{(A)}} + n_{S^{(Prev)}}$ is the total number of diagnostic features.

In addition, it must be noted that the set S can be represented by subsets of traditional S_t and alternative S_a diagnostic features $(S_t \neq S_a)$, i.e. $S = S_t \cup S_a, S_t \cap S_a = \varnothing$.

The elements of the set of diagnostic features S can be measured in different scales from the numerical scale (for example, the parameters of the structural elements of the BMS/I with the LCF) to the ordinal (for example, the degree of the disease manifestation from the previous examination data) and the nominal ones (for example, the presence of risk factors from anamnesis). Therefore, a measurement scale V_j is defined for each diagnostic feature s_j.

If the feature s_j is numeric (quantitative) then the set of values V_j of the numerical feature s_j is defined as follows: $V_j = \left\{ v \middle| v \in \left[v_{\min}, v_{\max} \right]; v_{\min}, v_{\max} \in \mathbb{R} \right\}$, where $v_{\min}, v_{\max}$ are the minimum and maximum values of a numerical feature s_j, i.e. the range limits of a numerical feature measuring; $\mathbb{R}$ is the set of real numbers. If the feature s_j is ordinal or nominal then the set of values V_j of the ordinal or nominal feature s_j is defined as follows: $V_j = \left\{ v_{jk} \middle| v_{jk} \in \mathbb{Z}, k \in \{1,2,\ldots,n_{V_j}\} \right\}$, where v_{jk} is the k-th value of diagnostic feature s_j; $\mathbb{Z}$ is the set of integers; n_{V_j} is the number of values for the diagnostic feature s_j. Then for all the diagnostic features S the set of values of diagnostic features V can be determined as $V = \left\{ V_j \middle| j \in \{1,2,\ldots,n_S\} \right\}$. Since the set of diagnostic features S can be represented as $S = S^{(X)} \cup S^{(A)} \cup S^{(Prev)}$ then the set of values of the diagnostic features V can also be represented as $V = V^{(X)} \cup V^{(A)} \cup V^{(Prev)}$, where $V^{(X)}, V^{(A)}, V^{(Prev)}$ are subsets of values of diagnostic features of BMS/I with LCF, anamnesis and previous examinations, respectively.

In order to bring the values of all diagnostic features V to one measurement scale, a set of value ranges (diagnostic ranges) $B_j = \left\{ b_{jk} \middle| k \in \{1,2,\ldots,n_{B_j}\} \right\}$ is defined for each of the diagnostic features s_j, $j = \overline{1,n_S}$, where b_{jk} is the k-th value range of the diagnostic feature s_j; n_{B_j}

is the number of value ranges for the diagnostic feature s_j. The setting of the allowable value range of a numerical feature s_j for diagnostic ranges b_{jk} corresponds to values of the type "norm", "below/above the norm", "boundary values", "dangerous values", "critical values", etc., characteristic of ordinal features and accepted in the given objective domain of medicine. The number of diagnostic ranges n_{B_j} can be different for different features. Then for all diagnostic features, the set of diagnostic ranges B can be defined as $B = \{B_j | j \in \{1,2,\ldots,n_S\}\}$.

Also in the mathematical model, the set of symptom-complexes is defined as $C = \{C_i | i \in \{1,2,\ldots,n_C\}\}$, where C_i is the i-th symptom-complex of a system of diagnoses $D = \{d_k | k \in \{1,2,\ldots,n_D\}\}$; n_C is the number of symptom-complexes; d_k is the k- th diagnosis; n_D is the number of diagnoses. A symptom-complex is an informative combination of the values of diagnostic ranges relative to the system of diagnoses D: $C_i = (b_{j_1 k_1},\ldots,b_{j_r k_l},\ldots,b_{j_n k_m})$, where $b_{j_r k_l} \in B_{j_r}$ are the values of diagnostic ranges; $j_r \in \{1,2,\ldots,n_S\}$ are the indices of diagnostic ranges included in the symptom-complex C_i; $k_l \in \{1,2,\ldots,n_{B_{j_r}}\}$ are the indices of values of diagnostic ranges from the set of B_{j_r}; $n_{B_{j_r}} = |B_{j_r}|$ is the number of value ranges in B_{j_r}.

To implement a biomedical DSS, the necessary condition is that the set of possible diagnoses D must form a complete group of incompatible events, i.e.

$$p\left(\bigcup_{k=1}^{n_D} d_k\right) = \sum_{k=1}^{n_D} p(d_k) = 1. \qquad (2)$$

The set of recommendations $R = \{r_l | l \in \{1,2,\ldots,n_R\}\}$, where r_l is the l-th recommendation list, must also form a complete group of incompatible events, i.e. $p\left(\bigcup_{l=1}^{n_R} r_l\right) = \sum_{l=1}^{n_R} p(r_l) = 1$.

Let the set $D = \{d_k | k \in \{1,2,\ldots,n_D\}\}$ be a universal set of diagnoses, and let there be given a family of characteristic functions $\mu_{\tilde{D}_j}(d_k)$, $j = \overline{1,n_{\tilde{D}}}$, that show the affiliation of the k-th diagnosis to the j-th diagnostic conclusion. Then the fuzzy subset $\tilde{D}_j = \{(d_k, \mu_{\tilde{D}_j}(d_k)) | d_k \in D\}$ corresponds to the fuzzy concept "the j-th diagnostic conclusion". In this case the characteristic function $\mu_{\tilde{D}_j}(d_k)$ takes a value from the linearly ordered set of accessories $M = [0,1]$.

Let us denote the set of fuzzy concepts (diagnostic conclusions) as $\tilde{D} = \{\tilde{D}_j | j \in \{1,2,\ldots,n_{\tilde{D}}\}\}$ (Filatova & Galkin 2012), where $\tilde{D}_j$ is the j-th fuzzy set of diagnoses; $n_{\tilde{D}}$ is the number of fuzzy diagnostic conclusions. In particular, if deterministic logic is used for diagnostics, then $M = \{0,1\}$ and the fuzzy set is considered as a classical one, at that $\tilde{D}_j = \{(d_j, \mu_{\tilde{D}_j}(d_j)) | d_j \in D, \mu_{\tilde{D}_j}(d_j) = 1\} = \{d_j\}$. In applying probabilistic logic, each diagnosis d_k is given by a conditional probability $p(d_k | C_j)$ which is the characteristic function $\mu_{\tilde{D}_j}(d_k)$, i.e. $\tilde{D}_j = \{(d_k, \mu_{\tilde{D}_j}(d_k)) | d_k \in D, \mu_{\tilde{D}_j}(d_k) = p(d_k | C_j)\}$. In the latter case, the fuzzy set $\tilde{D}_j$ is subnormal, since $\sup_{d_k \in D} \mu_{\tilde{D}_j}(d_k) \neq 1$, therefore, it must be normalised.

In addition to the sets discussed above, a set of correspondences $f = \{f_{XS}, f_{MrS}, f_{SV}, f_{VB}, f_{BC}, f_{CD}, f_{DR}\}$ is defined in the mathematical model M_M of the instrumental examination process (1). Let us consider in more detail the components of this set.

In the mathematical model M_M, there is a correspondence $f_{XS}: X \to S^{(X)}$ that relates the recorded BMS/I with LCF X to the subset of diagnostic features $S^{(X)}$: $f_{XS}(x_i[\cdot]) = \{s_j | s_j \in S^{(X)} \wedge (x_i[\cdot], s_j) \in X \times S^{(X)}\}$. Since the correspondence f_{XS} is everywhere definite, i.e. $D(f_{XS}) = X$ is the domain of f_{XS}, then the correspondence is a mapping, at that $R(f_{XS}) = S^{(X)}$ is the codomain of f_{XS}. The correspondence f_{XS} is not functional for the second component, since many diagnostic features of BMS/I with LCF are formed for each signal $x_i(\cdot)$. This mapping is realised by the procedure developed by the authors for the morphological analysis of BMS/I with LCF (Povoroznyuk et al. 2015). The stage of morphological analysis of BMS/I with LCF is one of the most important stages of instrumental examination, since errors at this stage lead to acceptance of erroneous diagnostic solutions or to refusal to make a decision in general. This stage requires the use of specialised methods of data morphological analysis taking into account the features of BMS/I with LCF and the methods of their transformation.

Also there is a correspondence $f_{MrS}: MR \to S$ that links patient data from a medical record with the set of diagnostic features S:

$$f_{MrS}(mr_i) = \left\{ s_j \middle| s_j \in (S^{(A)} \cup S^{(Prev)}) \wedge (mr_i, s_j) \in MR \times (S^{(A)} \cup S^{(Prev)}) \right\}.$$

The domain of f_{MrS} is $D(f_{MrS}) \neq MR$, since not all the patient data from the medical record correspond to the diagnostic features (for example, the patient's name or the date of the instrumental examination), so the correspondence f_{MrS} is not a mapping, at that $R(f_{MrS}) = S^{(A)} \cup S^{(Prev)}$ is the codomain of f_{MrS}.

The pair of correspondences f_{XS} and f_{MrS} allows to form the set of diagnostic features S.

The function $f_{SV}: S \to V$ sets a bijective mapping of the set of diagnostic features S onto the set of diagnostic feature values V: $f_{SV}(s_j) = \left\{ V_j \middle| V_j \in V \wedge (s_j, V_j) \in S \times V \right\}$.

The set of possible combinations of diagnostic feature values CP_V is a Cartesian product of sets V_j, $j \in \{1,2,\ldots,n_S\}$: $CP_V = V_1 \times \ldots \times V_{n_S} = \left\{ (v_1,\ldots,v_{n_S}) \middle| v_1 \in V_1,\ldots,v_{n_S} \in V_{n_S} \right\}$, where $|CP_V| = \prod_{i=1}^{n_S} |V_i|$ is the cardinality of the set CP_V. The set of possible combinations of diagnostic ranges CP_B is a Cartesian product of sets B_j, $j \in \{1,2,\ldots,n_S\}$: $CP_B = B_1 \times \ldots \times B_{n_S} = \left\{ (b_1,\ldots,b_{n_S}) \middle| b_1 \in B_1,\ldots,b_{n_S} \in B_{n_S} \right\}$, where $|CP_B| = \prod_{i=1}^{n_S} |B_i|$ is the cardinality of the set CP_B.

Then the correspondence $f_{VB}: CP_V \to CP_B$ defines a surjective and non-injective mapping of the set of tuples of diagnostic feature values CP_V onto the set of tuples of value ranges CP_B: $f_{VB}(cv_j) = \left\{ cb_k \middle| cb_k \in CP_B \wedge (cv_j, cb_k) \in CP_V \times CP_B \right\}$, where $cv_j = (v_1,\ldots,v_{n_S})$ is the j-th tuple made up of elements $v_1 \in V_1,\ldots,v_{n_S} \in V_{n_S}$; $cb_k = (b_1,\ldots,b_{n_S})$ is the k-th tuple made up of elements $b_1 \in B_1,\ldots,b_{n_S} \in B_{n_S}$. The mapping is not injective due to the fact that $|CP_V| \neq |CP_B|$, since in the general case $|V_i| \neq |B_i|$ (for example, for numerical features). In fact, the mapping f_{VB} is the transition from the space of diagnostic feature values CP_V into the space of diagnostic ranges CP_B.

Projections of the set CP_B on all possible combinations of coordinate axes form a set SP of all possible combinations of diagnostic ranges:

$$SP = \left\{ pr_i CP_B, pr_{i_1,\ldots,i_j,\ldots,i_q} CP_B \middle| i,j,q \in \{1,2,\ldots,n_S\} \right\} = \left\{ SP_i \middle| i \in \{1,2,\ldots,n_{SP}\} \right\},$$

where SP_i is the i-th system (tuple) of diagnostic ranges, formed by the corresponding projection; $n_{SP} \leq (2^{|CP_B|} - 1)$ is the number of possible projections.

Then it is possible to set the correspondence $f_{BC}: SP \to C$ of the set of possible combinations of diagnostic ranges SP onto the set of symptom-complexes C:

$$f_{BC}(SP_i) = \left\{ C_j \middle| C_j \in C \wedge (SP_i, C_j) \in SP \times C \right\}.$$

This correspondence is not a mapping, since only an informative combination of diagnostic ranges relative to the diagnoses system D is included in the set of symptom-complexes, i.e. $C = \left\{ C_i \middle| C_i \in SP, i \in \{1,2,\ldots,n_C\} \right\} \subseteq SP$, where C_i is the i-th symptom-complex relative to the diagnoses system D; $n_C \leq n_{SP}$ is the number of symptom-complexes.

The informativity (usefulness) of the diagnostic ranges B_j, included in the system SP_i, relative to the diagnoses system D, is determined in two ways:

1. by expert assessments;
2. by calculation of the informativity of the features using the training dataset.

In the first case, on the basis of their own experience, experts, who are doctors, determine the diagnostic value of each of the ranges included in the system SP_i. In the second case, the

concepts of entropy or quantities of information are used to determine the informativity of the diagnostic ranges included in the system SP_i.

Based on the statistical information obtained from the training dataset, it is possible to calculate the a priori probabilities $p(d_k)$ of each established diagnosis $d_k \in D$. Since condition (2) is fulfilled, the uncertainty of the system of possible diagnoses by definition can be estimated by binary entropy (Akhutin et al. 2002):

$$H(D) = -\sum_{k=1}^{n_D} p(d_k)\log_2 p(d_k), \quad (3)$$

where $H(D)$ is the measure of the uncertainty (entropy) of the system of diagnoses; $p(d_k)$ is the a priori probability of the k-th diagnosis d_k.

The entropy value $H(D) \geq 0$, and for equiprobable diagnoses, i.e. if $p(d_k)=\frac{1}{n_D}$ for $\forall k = \overline{1,n_D}$, the value $H(D)$ will be maximum, at that expression (3) takes the following form:

$$H(D) = -\sum_{k=1}^{n_D} p(d_k)\log_2 p(d_k) = -\sum_{i=1}^{n_D} \frac{1}{n_D}\log_2 \frac{1}{n_D} = \log_2 n_D.$$

The entropy value $H(D)$ as a measure of the uncertainty of the system of diagnoses D changes when new information enters the system in the form of values of diagnostic ranges obtained as a result of instrumental examination of a patient. In this case, the decrease in entropy occurs by an amount equal to the amount of information entered. The minimum entropy value $H(D) = 0$ and corresponds to a valid event.

The uncertainty of the system of possible diagnoses D can also be estimated by the amount of information that can be introduced by the system of diagnostic ranges SP_i. In this case, for the diagnostic range system SP_i and the system of diagnoses D, there is a valid inequality

$$I_D(SP_i) \leq H(D). \quad (4)$$

Expression (4) becomes equality only for a system of deterministic diagnostic ranges.

Then the amount of information entered into the system of diagnoses as a result of measuring the diagnostic ranges b_j included in the system SP_i is defined as the difference between the entropy value before and after the instrumental examination:

$$I_D(SP_i) = H(D) - H(D \mid SP_i), \quad (5)$$

where $I_D(SP_i)$ is the amount of information entered into the diagnosis system after measuring the feature ranges b_j included in the system SP_i as a result of an instrumental examination of a patient; $H(D)$ is initial (before the instrumental examination) entropy of the system of diagnoses; $H(D|SP_i)$ is the entropy of the system of diagnoses after an instrumental examination, taking into account the measurement of the diagnostic ranges b_j included in the system SP_i (conditional entropy).

The conditional entropy $H(D|SP_i)$ of the system of diagnoses D, provided that all diagnostic ranges b_j from the system SP_i are measured, is calculated as follows:

$$H(D \mid SP_i) = \sum_{j=1}^{n_{SP_i}} p(b_j)H(D|b_j), \quad (6)$$

where $H(D|b_j)$ is the partial conditional entropy of the system of diagnoses D, provided that the diagnostic range b_j from the system SP_i has been measured; n_{SP_i} is the number of diagnostic ranges in the system SP_i.

Taking into account the general definition of entropy (3), the partial conditional entropy $H(D|b_j)$ can be calculated as

$$H(D \mid b_j) = -\sum_{k=1}^{n_D} p(d_k|b_j)\log_2 p(d_k|b_j), \quad (7)$$

where $p(d_k|b_j)$ is a posteriori conditional probability of diagnosis d_k, provided that the diagnostic range b_j from the system SP_i has been measured.

Then, substituting (7) into (6), we obtain

$$H(D \mid SP_i) = -\sum_{j=1}^{n_{SP_i}} \sum_{k=1}^{n_D} p(b_j) p(d_k|b_j) \log_2 p(d_k|b_j).$$

Thus, according to expressions (5)–(7), the value $I_D(SP_i)$ based on the amount of information received characterises the diagnostic value of the system of diagnostic ranges SP_i in relation to the system of diagnoses D. At the same time, the system of the diagnostic range SP_i has a different informativity with respect to each diagnosis $d_k \in D$, i.e. for some diagnoses the system SP_i under consideration will be more informative than for others, and for some diagnoses it may not be informative at all.

Then the total informativity $I_D(SP_i)$ of the system of diagnostic ranges SP_i relative to the system of diagnoses D can be determined by means of the values of the private informativity of the system SP_i relative to the diagnosis d_k as follows: $I_D(SP_i) = \sum_{k=1}^{n_D} p(d_k) I_{d_k}(SP_i)$, where $I_{d_k}(SP_i)$ is the private informativity of the system of diagnostic ranges SP_i relative to the diagnosis d_k. At the same time

- $I_{d_k}(SP_i) > 0$, if the system SP_i has positive information (the values of the diagnostic ranges b_j included in the system SP_i confirm the diagnosis d_k);
- $I_{d_k}(SP_i) < 0$, if the system SP_i has negative information (the values of diagnostic ranges b_j included in the system SP_i, refute the diagnosis d_k);
- $I_{d_k}(SP_i) = 0$, if the system SP_i does not carry information relative to the diagnosis d_k.

In the instrumental examination of a patient, the diagnostic value of the system SP_i relative to the system of diagnoses D for uncorrelated diagnostic ranges b_j from the system SP_i is determined by the following expression:

$$I_D(SP_i) = \sum_{j=1}^{n_{SP_i}} I_D(b_j), \tag{8}$$

where $I_D(b_j) = \sum_{k=1}^{n_D} p(d_k) I_{d_k}(b_j)$ is the amount of information of the diagnostic range value b_j relative to the system of diagnoses D; $I_{d_k}(b_j)$ is the private information value of the diagnostic range value b_j relative to the diagnosis d_k.

Depending on the scale of measurement of the values of diagnostic ranges b_j, the private informativity $I_{d_k}(b_j)$ can be determined in various ways (Povoroznyuk 2011, Gubler 1978). If the diagnostic ranges b_j are correlated, the diagnostic value of the system of dependent diagnostic ranges SP_i relative to the system of diagnoses D is determined by the following expression: $I_D(SP_i) = I_D(b_1) + I_D(b_2 \mid b_1) + I_D(b_3 \mid b_2 b_1) + \ldots + I_D(b_{n_{SP_i}} \mid b_{n_{SP_i}-1} \ldots b_1)$, where $I_D(b_j \mid b_{j-1} \ldots b_1)$ is conditional private informativity. Since a sufficiently large number of diagnostic ranges is formed as a result of instrumental examination of a patient, certain difficulties arise in calculating $I_D(SP_i)$ by (4) related to the volumes and representativity of the training dataset (the so-called "curse of dimensionality"), therefore, even with strong correlation values of diagnostic ranges b_j, the diagnostic value $I_D(SP_i)$ is calculated by (8).

To verify the completeness of the description, when condition (4) is satisfied, we use the information completeness coefficient $k_{ic}(SP_i, D)$ of the system of diagnostic ranges SP_i with respect to the diagnosis system D, which is calculated as follows (Povoroznyuk 2011):

$$k_{ic}(SP_i, D) = \frac{I_D(SP_i)}{H(D)}. \tag{9}$$

Analogously, we calculate the coefficient of information completeness $k_{ic}(b_j, D)$ of each value of the diagnostic range b_j

$$k_{ic}(b_j, D) = \frac{I_D(b_j)}{H(D)}. \quad (10)$$

Since the process of measuring the values of diagnostic ranges is associated with time, material, financial and other costs, then taking into account the overall complexity of measuring the value of the diagnostic range b_j, it is possible to calculate the diagnostic value of coefficient $k_{dv}(b_j, D)$ by the following expression (Povoroznyuk 2011):

$$k_{dv}(b_j, D) = \frac{k_{ic}(b_j, D)}{k_{mc}(B_j)}, \quad (11)$$

where $k_{mc}(b_j)$ is the total assessment of the complexity of measuring the value of the diagnostic range b_j.

According to expression (11), the diagnostic value of the diagnostic range value b_j is greater the greater is its information completeness and the less the complexity of its measurement. Then for the system of independent diagnostic ranges SP_i, taking into account expressions (8), (9)-(11), the following relations are valid:

$$k_{ic}(SP_i, D) = \sum_{j=1}^{n_{SP_i}} k_{ic}(b_j, D); k_{dv}(SP_i, D) = \sum_{j=1}^{n_{SP_i}} k_{dv}(b_j, D),$$

where $k_{ic}(SP_i, D), k_{dv}(SP_i, D)$ are the coefficients of information completeness and diagnostic value of the system of diagnostic ranges SP_i relative to the system of diagnoses D, respectively.

Thus, the correspondence f_{BC} is determined by decisive rules that take into account the informativity, completeness of the description, compactness and consistency of the values of the diagnostic ranges included in the symptom-complexes $C_j \in C$.

Taking into account the obtained symptom-complexes in the mathematical model M_M, the correspondence $f_{CD} : C \to \tilde{D}$ of the set of symptom-complexes C onto the set of fuzzy diagnostic conclusions is $\tilde{D} : f_{CD}(C_j) = \left\{ \tilde{D}_k \middle| \tilde{D}_k \in \tilde{D} \wedge (C_j, \tilde{D}_k) \in C \times \tilde{D} \right\}$. In general, the correspondence f_{CD} is not a mapping of the set C onto the set $\tilde{D}$, since the elements of the set $\tilde{D}$ are fuzzy diagnostic conclusions; therefore, the same symptom-complex may point to different fuzzy diagnostic conclusions and several different symptom-complexes may point to one fuzzy diagnostic conclusion. The implementation of the correspondence f_{CD} is associated with the use of various diagnostic decision rules, adopted, for example, in decision theory or the theory of pattern recognition (Povoroznyuk 2011, Burtsev et al. 2013, Ignat'ieva et al. 2008).

Also, in the mathematical model M_M, the correspondence $f_{DR} : \tilde{D} \to R$ of the set of fuzzy diagnostic conclusions $\tilde{D}$ onto the set of recommendations R is given, and this correspondence is surjective and not injective. The mapping is non-injective because $n_D \geq n_R$, i.e. different fuzzy diagnostic solutions may correspond to the same recommendation.

Thus, the mathematical model of instrumental examination of patients is developed, which is used for the formalisation of knowledge formed as a result of the information processing, obtained also from the analysis of digital BMS/I with LCF.

3 CONCLUSIONS

1. A mathematical model of the instrumental examination process of patients is developed, taking into account the transformation stages of information obtained also from the analysis of recorded BMS/I with LCF, which made it possible to identify critical states of the instrumental examination.
2. Further research is focused on the development of the methods necessary to implement the relevant stages of information processing in biomedical decision support systems.

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