

# **COLUMN FLOTATION MACHINE – INNOVATIVE AERATION, VIBRATORY – ACOUSTIC AND TECHNOLOGICAL RESEARCHES**

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## **1. Introduction**

During the past years a great efforts has been paid for improvement of the flotation process both mechanical equipment and the chemical interactions reagent – mineral. An important direction in this area is column flotation. The industrial experience shows that implementation of column flotation leads to decrease of the number of cleaning operations due to the improved selectivity.

G.M. Callow patented the first pneumatic flotation cell, which used air sparging through a porous bottom and horizontal slurry flow, in 1914. The first countercurrent column flotation device was designed and tested by Town and Flynn in 1919. Cross-current pneumatic flotation machines were widely used in industry in 1920's and 1930's, but were later replaced by the impeller-type flotation devices in mineral processing plants. Dissolved-air flotation became the main type of flotation for water treatment applications. These substitutions were the result of the absence of effective and reliable air spargers for fine bubble generation and by the lack of automatic control systems on the early columns. During this period, both the poor flotation selectivity and entrainment of slimes characteristic of impeller-type cells was offset by the use of complex flowsheets using large numbers of cleaner stages and recycle lines. Column flotation devices were re-introduced for mineral processing in the late-1960's in Canada by Boutin and Wheeler (1967) at which time wash water addition to the froth was used to eliminate entrainment of hydrophilic materials to the float product. By the late-1980's column flotation had become a proven industrial technology in the mineral industry. These separators are routinely used on their own or in conjunction with other types of devices within separation circuits

The flotation technique and technology developments are closely connected with the improvement of the flotation machines design.

The main road solving the task for flotation process intensification is the creation of high – productive flotation machines.

The contemporary requirements in the field of pulp aeration and bubble mineralization theories impose the design of pneumatic flotation machines of column type that find application at the flotation of ores of non-ferrous, noble, rare and ferrous metals, coal and other mineral resources as well as wide use at purification of waste water from different productions. (Chernykh, 1996)

The column flotation machine wide practical introduction appears to be an important achievement during the last decade in the field of flotation concentration. The advantages of the column flotation machine in comparison with the mechanical and pneumomechanical are the following: increased productivity improved hydrodynamics, of concentrate with the required quality at minimal quantity purifying and control operations, production area decrease and economy of electric power.

The movement nature of the particle and the bubble is an important factor, which determines the probability of flotation complex formation, mineralization rate, flotation speed and the process power consumption. (Chernykh, 1996) The inertial forces which destroy the particle - bubbles complex in column are insignificant. This is connected with the absence of a stirring device and the pulp flow low turbulence.

The increase of the air bubbles flotation activity is connected with the increase of their conditioning time, i.e. the interval between the formation moment and the bubble mineralization. In consequence of the column considerable height the sojourn duration of the air bubbles in it is not longer than 20s, i.e. a mineralization process at optimal flotation activity of the bubbles is realized. The basic research field in column devices is the fine-grained pulp flotation. The absence of intensive pulp mixing, product purification zone, high position of the feeding level, and the thick froth layer contribute to the obtaining of concentrate of better quality in comparison with the impellers. (Rubbinshtain, 1989).

There is a bigger probability of capture and withstanding of the coarse particle to the bubble so in some cases the column flotation machine use could turn out to be expedient for coarse – grained material flotation, as well. (Foot, 1986)

The flotation machines work effectiveness depends on the conditions of the air dispersion. The aerators must provide a maximum gas content at an optimal average massiveness of the bubbles. The following claims are demanded to the aerators: providing for such a bubbles size that assures the flotation complex emerging, minimal pulp macrocirculation in the chamber, stable aeration characteristic.

The aerators designs developed in accordance with the requirements could be combined by the action principle as follows: pneumatic, hydraulic (swift - flowing) and pneumohydraulic.

The basic aerator type in the column flotation machine is the pipe device. An aerator representing a set of tightens in group hollow rings from porous (wool or porous polyethylene) or elastic (raw rubber) material is proposed for the aeration characteristics improvement. Changing the group tightening extent regulates the bubbles dispersive composition. The pulley aerator design is analogous. The ring pivot is vertically situated and the diameter increases from top to bottom. As a separate tendency the vibrating pneumatic flotation machines design should be mentioned. Their action principle is based on air dispersion by vibrations at its feeding to the aeration chamber. The pulp vibrating turbulent movement creation reinforces at the liquid flow through special lattices.

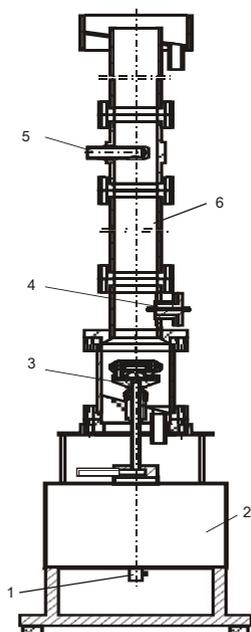
## **2. Experimental part**

Dedelyanova and Metodiev (2002) presented a laboratory model of a vibratory column flotation machine constructed in the Laboratory of “Vibro-acoustic Intensification of the Technological Processes” at the Department of Mineral Technologies of the University of Mining and Geology “St. Ivan Rilski”, Bulgaria.

It is realized on a modular principle, which provides a possibility for determination of the feeding device optimal height infinitely – variable regulation of the pulp level and precise determination of the necessary quantity of extra water for the froth layer irrigation. The vibratory column flotation machine basic elements are sensor (fig. 1-1), vibrator (fig. 1-2), air disperser (fig. 1-3), module for creation of single gas bubbles (fig. 1-4), feeding device (fig. 1-5), machine chamber (fig. 1-6).

The flotation batch tests were carried out in a laboratory vibratory column flotation unit – two meters high and 50 mm in diameter. It is

made of transparent plastic tube with internal volume 4.2 l. The sample for the tests is rougher copper concentrate. Due to the fact that the sample is already conditioned reagent collector is not used, only frothier – from 15 to 25 drops in order to a high froth layer to be obtained.



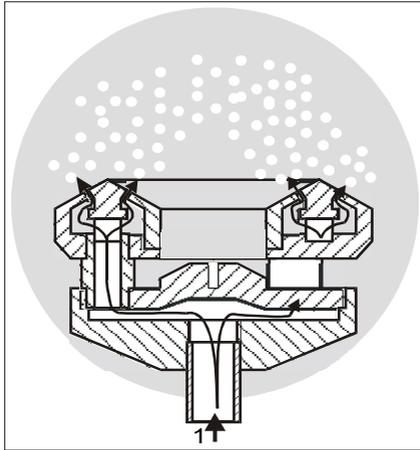
**Fig. 1.** Vibratory column flotation machine – scheme of principle

The vibratory disperser provides the opportunity of certain technological parameters research: gas bubble size change, change of the gas bubbles emerging speed and solid phase sedimentation speed, the influence on the opportunity of mineral particles attachment to the air bubbles, purifying of the froth layer from rock particles.

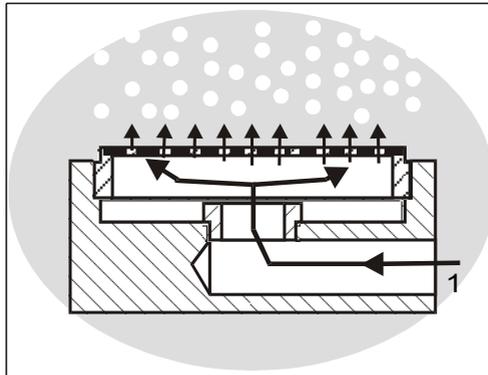
The vibrator is electrodynamic type, power 40 W and a possibility of a wide range frequency and amplitude change, overlapping the supposed range of vibratory parameters, required by the research.

Two modifications of the air- dispersing unit were constructed- for vertical and horizontal vibrations. The air-dispersing unit for vertical vibrations (fig.2-1-air) disperses air by its transition through an annular slot, which allows rate regulation within certain range and

production of gas bubbles of certain size by regulation of vibrations frequency and amplitude.



**Fig.2.** The air-dispersing unit for vertical vibrations



**Fig.3.** The air-dispersing unit for horizontal vibrations

Simultaneously, with the bubbles production, the vertical oscillations create a vibroacoustic field, which is distributed along the column height. The air-dispersing unit for horizontal vibrations (fig.3-1-air) produces gas bubbles through round holes, perpendicular to the direction of the vibratory field vector.

The feeding modulus allows pulp supply into the flotation machine and even distribution of pulp along the column section as well as filling with water for vibroacoustic measurements.

The commonly used quantity of air in the tests is 100 l/h.

The level of the pulp is adjusted based on the principle of connected vessels by vertical movement of overflowing ring. The tailings are discharged by plastic pipe and valve into a vessel.

The formed complexes particle-bubble are floating into the froth layer - about  $30\text{-}40\cdot 10^{-2}$  m high. The counter flow of fresh water is not necessary because the vibrating field cleans the complexes during their upward movement enough efficiently. The froth is collected also into a vessel for drying and assay. The time of flotation is 15 to 30 min. depending on the solids content. The tests are carried out at solids content of the slurry: 0,50, 0,51, 0,75, 1,10,1,50, 1,70, 2,10%, 2,50, 2,70, 2,90, 3,10, 3,80, 5,30, 5,90, 6,60, 10,30%.

The energy of vibrations could be characterized by the pointer –  $Af^2$  and their influence upon flotation process is evaluated based on the difference in the grade and recovery of the valuable component into the concentrate compared to the zero test. The flotation tests were carried out at frequencies from 25 to 40 Hz, amplitudes 2.0, 2.5 and 3 mm and values of  $Af^2$  respectively 1500, 1800, 2250, 2450, 3200 and 3675.

### **3. Results and discussion**

The use of vibration impact on flotation process is based on the effects, caused by vibrations onto a heterogeneous media – effective decrease of viscosity of the liquid phase, vibrating of particles of the solid phase, periodic change in the shape of the gas bubbles. These effects displayed at the original design of column flotation machine having a big ratio between height and cross section diameter, should be determined at definite values of vibrations' frequency and amplitude so that the resonance phenomena to be displayed into the chamber. The batch tests carried out at this area should give the answer to the questions concerning the definite values of the effective interval of vibrations' frequency and amplitude for practical use in technological tests with definite products of the mineral processing industry.

During the tests are performed two types of tests: measurement of flotation speed of unique air bubble in calm and vibrating liquid me-

dia and calculation of the difference in their speed; measurement of the amplitude of vibrations along the column flotation machine as a function of the water height. These tests are carried out at different frequencies and amplitudes of vibrations of the vibratory-acoustic disperser.

### **3.1 Definition of the vibrations influence upon the speed of gas bubbles floating**

The stream-lining character of the bubbles is substantially different to the stream-lining of the solid phase in the liquid. The smallest bubbles at  $Re < 1$  keep their shape close to the spherical one because it is determined by the surface tension and the speed of flotation is defined by the Stock's law.

With increase of the bubbles diameter begins a process of deformation. This deformation violates the straightforward character of movement and changes its flotation speed.

The speed of flotation of the bubbles with deformed surface is bigger, which could be explained with the fact that thanks to the mobile border surface, the gradient of speeds in the liquid around the bubble are smaller than around the spherical one.

The decrease of speeds gradient leads to decrease in energy of dissipation in the liquid. The natural frequency of bubbles with diameter 1-4 mm is at the range of 200-500 Hz.

Therefore, the frequencies that have been researched at vibrating flotation are at the under-resonance range where stable amplitude of the bubbles vibration can be observed.

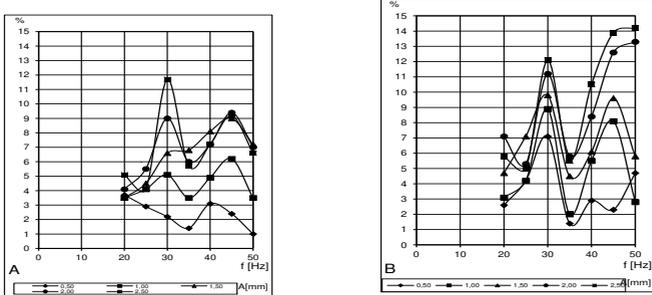
The general results of experimental determination of the flotation speed of gas bubble in vertical vibrating field are shown at the table 1.

The speed of flotation of gas bubble without vibrations is bigger than if they are applied. The decrease of speed depends on the amplitude of vibrations - with increase of amplitude the flotation speed decrease reaching to 13-14 % for bubble with diameter 4 mm. At the chart of these speeds at figs. 5 and 6 a sinusoidal dependence as a function of the frequency is definitely observed - the maximum is at 30 and 45 Hz and the minimum at 20 and 35 Hz.

Table 1

Velocity of bubble flotation without and with application of vibrations  
at water column 1.2 and 0.9 m and size – 4 mm

F, Hz	A, mm	V, m/s without vibrations	V, m/s with vibrations	Decrease in %	V, m/s without vibrations	V, m/s with vibrations	Decrease in %
20	0,5	26,43	25,21	3,7	26,55	25,86	2,6
	1,0	25,92	25,26	3,5	26,87	25,71	3,1
	1,5	26,43	25,26	3,5	26,47	25,28	4,7
	2,0	26,20	25,00	4,5	26,39	24,66	7,1
	2,5	26,30	24,84	5,1	26,39	25,00	5,8
25	0,5	26,14	25,42	2,9	26,47	25,42	4,2
	1,0	26,09	25,10	4,1	26,63	25,42	4,2
	1,5	26,32	25,00	4,5	26,55	24,66	7,1
	2,0	26,26	24,74	5,5	26,47	25,14	5,3
	2,5	26,03	25,10	4,1	26,32	25,21	5,0
30	0,5	26,49	25,59	2,2	26,55	24,66	7,1
	1,0	26,20	24,84	5,1	26,63	24,19	8,9
	1,5	26,20	24,44	6,6	26,55	23,94	9,8
	2,0	25,97	23,81	9,0	26,55	23,56	11,2
	2,5	26,09	23,12	11,7	26,39	23,32	12,1
35	0,5	26,61	25,81	1,4	26,87	26,16	1,4
	1,0	26,32	25,26	3,5	26,71	26,01	2,0
	1,5	25,97	24,39	6,8	26,55	25,35	4,5
	2,0	26,03	24,59	6,0	26,55	25,00	5,8
	2,5	26,03	24,69	5,7	26,47	25,07	5,5
40	0,5	26,32	25,37	3,1	26,71	25,77	2,9
	1,0	26,26	24,90	4,9	26,39	25,07	5,5
	1,5	26,14	24,05	8,1	26,63	24,73	6,1
	2,0	26,20	24,29	7,2	26,47	24,32	8,4
	2,5	25,97	24,29	7,2	26,47	23,75	10,5
45	0,5	26,55	25,53	2,4	26,87	25,94	2,3
	1,0	26,14	24,54	6,2	26,63	24,39	8,1
	1,5	26,09	23,81	9,0	26,63	24,00	9,6
	2,0	25,92	23,72	9,4	26,32	23,20	12,6
	2,5	26,03	23,76	9,2	26,32	22,84	13,9
50	0,5	26,67	25,92	1,0	26,63	25,28	4,7
	1,0	26,26	25,26	3,5	26,63	25,79	2,8
	1,5	25,97	24,29	7,2	26,55	25,00	5,8
	2,0	25,92	24,34	7,0	26,32	23,02	13,3
	2,5	25,92	24,44	6,6	26,32	22,78	14,2



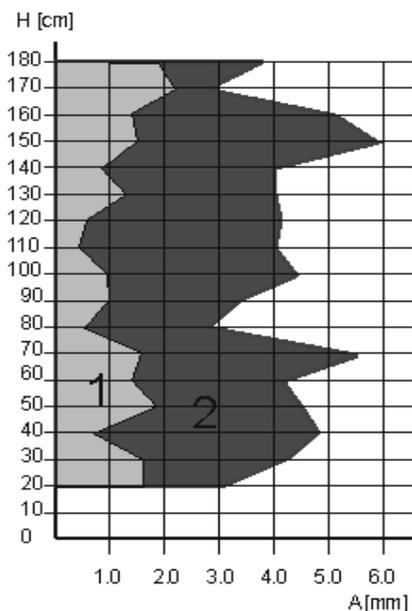
**Fig. 4.** Decrease of the speed of flotation of the bubbles in % as a subsequence of applied vibrating field: *A* - Height of the water column  $H=1.2$  m and *B* - Respectively  $H=0.9$  m

The decrease of flotation speed of the air bubbles in vibrating field is definitely connected with the amplitude of applied vibrations and is defined by the deformation of the bubble as a result of the processes of increase and decrease of pressure in the water column because of the pass of the wave processes. As a difference to the deformation of the bubbles during their free floating without vibrations, the deformation after application of vibrations is towards elliptical because of changeable pressure at the liquid. This pressure acts in vertical direction and diminishes the vertical size of the bubble enlarging the horizontal one, actually, the spherical bubble is changed into ellipsoid. The resistance to its floating is increased because of its increased horizontal diameter.

This behavior of the bubble ensures increase of the time for movement at the active zone of machine and increase the probability for contact with the particles of the solid phase. The contact bubble-particle is facilitated because of the difference in the vibration phases of the bubble and the solid particles and also makes easier the destruction of the hydrating layers which benefits the complex bubble-particle floating to the froth layer.

### 3.2 Hypothesis for the vibrations influence upon the speed of floating of gas bubble and elementary act of flotation

The smaller amplitude of vibrations at aeration of the water is explained by the loss of energy spent on change of the gas bubbles' shape. The changeable pressure caused by the front of the sound wave spreading along the height of column is the reason for resonance effects and on the other hand this pressure changes the shape of the bubbles from spherical to ellipsoidal with shorter vertical diameter and bigger horizontal one. This change in the shape explains the reason for the lower speed of flotation of the gas bubbles. The bigger diameter and the smaller height of the bubble lead to increase of resistance to the floating movement.



**Fig. 5.** Amplitude of vibrations along the height of column - 1 - with air bubbles; 2 - without them

The change in the shape of the bubbles is doing with the applied frequency - about 25-40 Hz. The decrease of the flotation speed practically increase the content of the gas phase and possibility for mineralization. The change of the diameter increases the possibility for contact with the sold phase during the process of mineralization — when the diameter increase the possibility for capturing of particles also increase. The vibration of the gas bubble determine the possibil-

ity for rejection of solid particles with hydrophilic surfaces. During the vibration of the bubble the perimeter of wetting is changing due to the changes of its diameter, leading to decrease of the capturing force and respectively the rejection of the hydrophilic particles.

The module for gas bubbles production provides the opportunity of air micrometric supply through changeable air nozzles of certain initial diameter. The pulp level is infinitely - variable regulated by the communicating vessels method.

#### **4. The vibrations influence on the solid phase**

Researches were carried out for the effect of frequency and amplitude of vibrations on speed of precipitation of mineral grains of different density. It was known that speed of precipitation decreases with the increase of intensity of oscillating, however the question of effect of diameter and relative weight of particles was still open. For that purpose a series of experiments were carried out with single mineral grains of a diameter from 0,09 to 0,155 and densities of different grains 2,65; 5,1 and 7,8 g/cm<sup>3</sup>. Frequency of oscillation changes from 20 to 70 Hz, and amplitude - from 1,5 to 3,0 mm. Average results of experiments were presented in tables 2. Experimental researches involved the conclusion that vibrating media provided additional force of resistance, applied to the mineral particle, which provoked decrease of the speed of falling down. This force is a function not only of frequency of applied vibrations, where experimental values showed decrease of speed depending on density of particles. It was visually observed that when the particle did not reduce significantly its speed/for certain vibration parameters/, its amplitude of oscillation is approximately equal to the amplitude of oscillation of the liquid. When speed of a particle reduces significantly its amplitude increases visibly.

This dependency between speed of falling down of mineral grains into a liquid vibrating medium and parameters of vibrating field may be explained as an interaction between two oscillation motions - motion of mineral grains and motion of applied vibrations.

For lower frequencies of vibration reduction of speed is lower due to lower vibration speed. For particles of higher density reduction is higher due to higher speed, with which they meet the pulsing medium.

Table 2

Decrease of the speed of precipitation in dependence of density of particles

Frequency f [Hz]	Amplitude 2 mm, d=0.12 mm		
	Reduction of V [%] for density g/cm <sup>3</sup>		
	2,65	5,1	7,8
	%	%	%
20	1,54		
30	3,39	5,38	4,1
40	1,34	7,6	9,73
50	4,54	6,59	7,68
60	5,6	6	8,77
70	4,22	9,6	12,16

#### 4.1. Experimental Research

Dedelyanova (2004) presented the results of research with vibratory column flotation machine with copper ore (cake taken from a control thickener).

The experiments have been carried out in a vibratory column flotation machine laboratory model with height 2000 mm and diameter 50 mm and effect of vibrations  $Af^2$  1500, 1800, 2250, 2450, 3200, 3675, respectively, at a frequency of vibration from 25-40 Hz and an amplitude of 2.0, 2.5 and 3.0 mm.

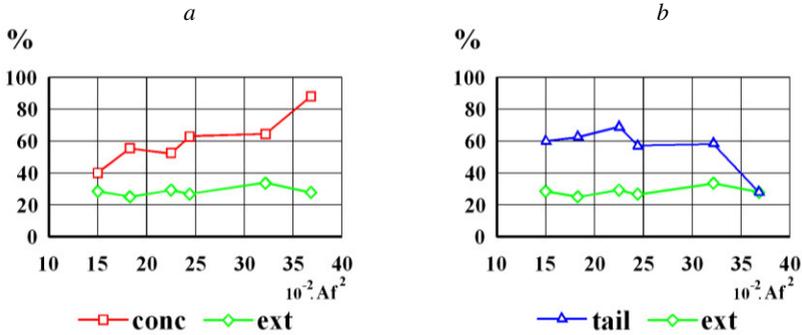
Dispersion of air is through a vibration dispersant designed for vertical vibration. Along with the creation of bubbles, vertical oscillations create a vibro-acoustic field, which is distributed along the height of the flotation machine camera.

At values of vibration effect  $Af^2=3200$  ( $f=40$  Hz,  $A=2.0$  mm) the copper content of the cake reaches 32.23% at a copper content in the sample output of 22.76%.

The content of copper in concentrate in the vibration increases 312%. The results are illustrated in Figure 6.

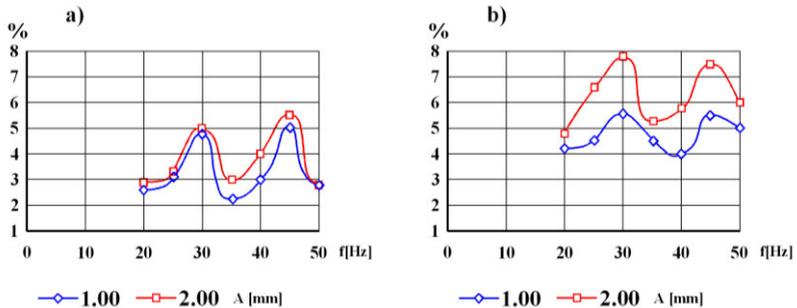
In summary, the effect of vibration  $Af^2=3675$  ( $f=35$  Hz,  $A=3.0$  mm) and solids in the pulp 3.8%, that value is highest for this flotation experiments, extraction is highest at 90.68% and value of copper content in concentrate is also highest at 31.44%.

The copper content in concentrate increases 3-3.5%, and copper output from 23.31-27.89%.



**Fig. 6.** *a* - dependence of copper content in concentrate and extraction from characteristic  $Af^2$  - effect of vibrations; *b* - dependence of copper content in tail and extraction by characteristic  $Af^2$

The vibratory effect on the gas bubble is applied after it detaches from the nozzle. The measurements results are presented in Figure 7. The gas bubble emerging velocity without vibrations is higher than that with vibrations. Vibrations contribute to the decrease in gas bubble emerging velocity and extend its stay in the column flotation machine.



**Fig. 7.** Decreasing of emergence velocity of gas bubble in % from emergence velocity without vibrations: *a* - for the height of the water column  $H=120$  cm and  $d = 3.5$  mm; *b* - for  $H=90$  cm and  $d=3.5$  mm

According to Bogdanov (1990), forces acting on the bubble and the particles attached to it, accounted on the vertical axis are two types: forces acting on the particle and forces caused from the attachment of particle to the bubble. The first type of forces are gravity of particle and ejection Archimedean force, as follows: gravity

$P = -\pi d_p^3 \rho_p g / 6$ , where  $d_p$  is aligned particle diameter,  $\rho_p$  is the density of the particle and  $g$  is the gravity acceleration; ejection Archimedeian force  $F_A = \pi d_p^3 \rho_l g / 6$ , where  $\rho_l$  is the density of the fluid displaced by the particle. The second type of forces are the Laplace force  $F_L$  and capillary force  $F_k$ : Laplace pressure force that is acting on the upper wall of the particle is  $F_L = -\pi d_p^2 \Delta p / 4$  where  $\Delta p = p_g - p_l$  is the Laplace pressure,  $p_g$  and  $p_l$  are respectively pressure in the gas and pressure in the liquid on the level of contact; capillary force  $F_k = \pi d_p \sigma \sin \theta$ , where  $\sigma$  is the surface tension recorded per unit length of contour of tangent wall of the particle and  $\theta$  is the angle between the tangent wall of the particle and the surface of the bubble. The resulting equilibrium equation of bubble and attached particle is  $P + F_A + F_L + F_K = \frac{\pi d_p^3 \rho_p a_l}{6}$ , where  $a_l = A \omega^2 \sin \omega t = 4\pi^2 A f^2 \sin \omega t$  is the acceleration of the liquid, due to the harmonious movement. To determine the angle  $\theta$  the equilibrium equation is expressed in the form

$$\sin \theta = \frac{d_p^2 g (\rho_p - \rho_l)}{6\sigma} + \frac{d_p \Delta p}{4\sigma} + \frac{2\pi^2 d_p^2 \rho_p A f^2 \sin \omega t}{3\sigma}. \quad (1)$$

The ratio of the hydrostatic pressure on the contact side of the particle to the Laplace pressure is defined as a form constant to the bubble near the particle.

$$\beta = \frac{\rho_l g d_b}{\Delta p} = \frac{\rho_l g d_b^2}{4\sigma} = \frac{\rho_l g d_0^2 \left( \frac{p_0}{p_0 + 4\pi^2 \rho_l A f^2 \sin \omega t} \right)^{\frac{2}{3}}}{4\sigma}. \quad (2)$$

From the equation (5) it follows that the size of the constant harmonic variation  $\beta$  is a function of effect of vibrations  $A f^2$ .

## 5. Collision efficiency in vibration

Productivity of flotation devices is determined by the flotation rate  $(d\varepsilon/dt) = f'(t)$ . For the case where the properties of the material in flotation conditions are constants ( $\varphi = \text{const}$ ,  $N = \text{const}$ ) then the equation assumes the shape  $(d\varepsilon/dt) = k(1 - \varepsilon)$ . According to this equation, the flotation velocity is proportional to the mass of flota-

tion material  $1-\varepsilon$  and is characterized by a probability  $k$  for flotation by one unit time. The solution of equation is  $\varepsilon = 1 - e^{-kt}$ , which determines the nature of the flotation process, reporting in time.

To obtain an integrated evaluation of the flotation machine efficiency, we must introduce the concepts of collision efficiency  $E = (M_0 / M)$ , where  $M_0$  is the mass of particles that meet with bubbles and  $M$  is the mass of all particles that are located in the level of contact at a point in time  $t$ .

In (Ralston, 1999), the parameter  $k$  is called the flotation rate constant and is regarded as proportional to the defined collision efficiency  $E$ .

In Rubbinshtain, (1989) the parameter  $E$  is called the coefficient of grip. For two flotation processes labeled 1 and 2 for particles of different sizes and ceteris paribus, is presented the dependence:

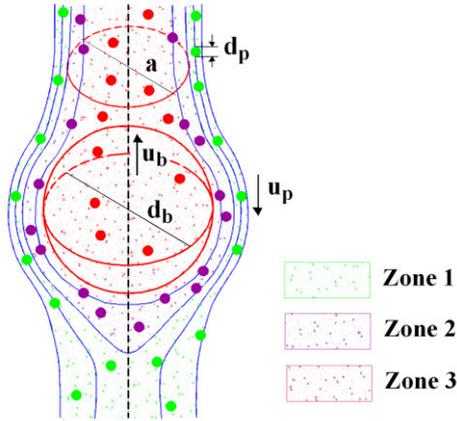
$$\frac{\ln(1 - \varepsilon_1)}{\ln(1 - \varepsilon_2)} = \frac{E_1}{E_2} . \quad (3)$$

Rubinschayn (1989) presented multiple formulas determining the number of collisions  $N$  and collision efficiency  $E$ , divided in two groups:

- 1) a relatively small group - formulae derived by accepting the model of turbulent motion of bubbles and particles;
- 2) formulae derived by analyzing the deterministic description of the bubble and particle movement, processes of interaction and relevant forces governing these processes.

According to Derjaguin and Dukhin (1960), 3 zones can be defined around the bubble (Figure 8):

- Zone 1 – outermost part – of hydrostatic interaction;
- Zone 2 (of attachment) – area of effect of surfaces;
- Zone 3 - of stability efficiency of the bubble - particle aggregate



**Fig. 8.** Scheme of collision of the particles with the bubble

In accordance with the areas of interaction in the aggregate bubble - particle (zone 1 and zone 2) the effectiveness of a collision can be represented by  $E = E_{il} + E_{vl}$ , where  $E_{il}$  is efficiency of laminar (gravitation) flow (ideal liquid) and  $E_{vl}$  is efficiency of viscous flow (viscous liquid). Taking into account the formulas presented in (Bogdanov, 1990; Rubbinshtain, 1989) with some approximations, one can accept the formulae.

$$E_{il} = \frac{G}{1+G} \left( 1 + 3 \frac{d_p}{d_b} \right) \text{ and } E_{vl} = \frac{3}{2} \left( \frac{d_p}{d_b} \right)^2 f, \text{ where } G = \frac{u_p}{u_b}, f \in [0.1, 1]. \quad (4)$$

From the ways of expressing the effectiveness of conflict, we can conclude that the vibrational motion of bubbles and particles is affected by the effectiveness of a collision. The impact is both through variable diameters of the bubbles and by the rate of ascent of the bubbles. Given that the collision efficiency is a dimensionless quotient of the ratio of the mass of the particle meeting the bubble and whole mass of flotation material, it is clear that we can not directly use formulae presented in the literature in the case of vibratory column flotation machine.

## 5.1. Determining the effectiveness of a collision at vibration impact

To determine the increasing of effectiveness of collision in the presence of vibration, we will use another approach. We will consider the model given by Bogdanov (1990) and supplement it with vibrational deformations of the bubble. We will also use results of experimental studies of Dedelyanova (2004).

For an analysis of the collision process, we accept a bubble as a sphere and we consider a rotating body around a vertical axis, passing through the center of the sphere. We denote by  $d_b$  the aligned bubble diameter,  $d_p$  is the aligned particle diameter, and  $a$  is the diameter of the cylindrical areas include zones 1 and 2 before the level of collision.

The cross section of the cylindrical region in diameter  $a$  is  $S_0 = \pi a^2 / 4$  and at the level of collision  $S_b = \pi (d_b + d_p)^2 / 4$ . Produced

for the effectiveness of conflict  $E = \frac{S_0}{S_b} = \left( \frac{a}{d_b + d_p} \right)^2$ . The parameter

$E$  is a ratio from the section of the straight part of the area, including zones 1 and 2 to section at the center of the bubble. Because the bubble vibrates with sound frequency  $f$ , then the mean diameter  $d_b$  is the same as without vibration. Performing a harmonic cycle of bubble volume maintains the transition from sphere to rotating ellipsoid. When the bubble diameter is  $d_b = 4 \text{ mm}$  and amplitude  $A = 0.7 \text{ mm}$  extremes on the vertical axis of the ellipsoid are  $z_{\min} = 3.3 \text{ mm}$ ,  $z_{\max} = 4.7 \text{ mm}$  with horizontal axes, respectively  $x_{\max} = y_{\max} = 2.48 \text{ mm}$  and  $x_{\min} = y_{\min} = 1.72 \text{ mm}$ . Due to the high velocity of the bubble oscillation at a given frequency  $f$  parameter  $a$  on Figure 8 reaches the diameter of the bubble  $d_b$ .

## 6. Conclusions

Although the basic concept of flotation column looks relatively simple, but the fundamental principles related to performance of flotation column are quite complex. The type of the relative motion of particles and bubbles is a major factor governing the probability of

bubble/particle attachment, bubble loading, flotation rate and power requirement of the processes.

Experimental research led to the conclusion that vibrating media provided an additional force of resistance applied to the mineral particle, which provoke a decrease of the speed of descent. This force is a function not only of the intensity of the vibrations of a complex variable vibration, but also of the frequency of vibration.

The use of vibrations enables the diameter of gas bubbles to be adjusted in dependence of the influence of vibrations parameters leading to a better dispersing of the gas phase at the column flotation machine. The decrease of floating speed of the gas bubbles ensures increase of the time for mineralization and also the probability for collision with the particles of the solid phase. The vibration of the gas phase also enables mechanically entrapped particles to be released.

As a logical consequence follows that at vertical vibrations the air disperser can disperse sufficient air quantity for the flotation process realization. The gas flow velocity decrease combines with the higher effective product content in the concentrate compared with that in the initial product that could be explained by the specific vibratory influence on the three-phase system.

From the analytical study of the collision efficiency are confirmed experimentally proven facts. It provides possibility to increase the residence time of the bubble in the zone of contact with the mineral particles. The vibrations of the hydrated layer allow for cost-effective implementation of the elementary act of flotation by providing larger relative velocities at the contact point between the bubble and the solid phase.

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