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THE STUDY OF ORE BREAKAGE IN BALL MILL TO ASSESS THE ENERGY EFFICIENCY OF ITS GRINDING

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Abstract

The results of mathematical modelling of ore breakage by a ball mill based on mathematical models of loaded and unloaded elastic elements are presented. The property of the arrangement of balls in the drum due to segregation is used, under the influence of which they are distributed with increasing size from the loading to the unloading throat, and in the cross-sections from the lining to the axis. It is shown that it is advisable to use the balls in the cross-section of the drum, where their size is 50 mm. The more the volume of ore is destroyed, the more the ball's energy is spent on this, the less energy will be converted into deformation of the elastic element. Consequently, the value of the deformation of the elastic element corresponds to the volume of the destroyed ore, as well as the energy efficiency of its grinding. The approach of interaction of balls with an elastic element from the point of view of mass phenomena will be more effective. The average value of deformations by all balls that have passed through the elastic element, arbitrarily accurately characterizes the average value of the destroyed volume of ore, that is, the energy efficiency of its grinding. The controlled parameter is found in accordance with the proposed relationship with the measured average deformation value of the elastic element. The deviation of certain values from the reference values is within 1%, which meets the requirements of the technological process.

Keywords: energy efficiency, mathematical modelling, ball mill, an elastic element, deformation

1. Introduction

Today, the reserves of rich iron ores are almost exhausted. Therefore a steady transition to the development of low-grade reserves with an iron content of 20-35% is taking place, with an increase in the content of the useful component to 65% in the beneficiation process. This trend, in particular, is noted in the priority areas outlined in [1]. Poor iron ores are crushed in several stages before enrichment. This process requires a large amount of electricity, grinding media, and lining. Especially high costs fall on the first grinding stage, which is carried out in ball mills. As a result, the cost of magnetite concentrate is growing, which is a problem of iron ore industries.

2. Actuality of the paper

This problem can be solved in various ways, one of the most effective is the grinding of the initial ore of the beneficiation plants in conditions of high energy efficiency of its grinding in ball mills. Under the highest energy efficiency of the grinding of the material should be understood the best use of the energy of the falling ball. If there is not enough material at the moment of impact under the ball, then a part of the acquired energy will not be useful. Besides, excess energy will be spent on the destruction of both the grinder and the lining. With an excess of ore under a falling ball, its breakage will not be effective. The technological unit will enter the emergency state of overload, which is not permissible. For any technological type of ore, it is possible to select such a volume of ore, which is subject to grinding, when the energy of the falling ball practically corresponds to the required energy for breakage. A specific small excess of the energy of the ball after the breakage of the ore can be an indicator of the energy efficiency of grinding. In these conditions, there is no waste of electricity, balls and lining. Scientists and practitioners have been working on solving this problem for a long time. As a result of the complexity of the processes in the ball mill, indirect methods of controlling technological parameters were used in the studies. For example, in [2], it is noted that the average power of the electric motor and the noise of the mill are used as a criterion for automatic optimization in extreme systems supporting the operating point on the static characteristic of the average power or intensity of the acoustic signal as a function of filling the drum near the extremum. The control action is the flow of the initial ore into the mill. In industry, such systems are not widely used, because the maximum performance by the finished product does not correspond to the operating point in the area of extremum. In this regard, the problem of automatic optimization of ore preparation is solved by compensating of disturbing influences of initial feed (which is problematic) or the search and justification of technological parameters, which are automatically controlled and directly characterize the energy efficiency of the breakage process in the mill drum.

Thus, [3] showed that there is a definite relationship between power consumption and filling of a ball mill drum with pulp. However, it is still difficult to use such a relationship. The work of [4], is also based on measuring the energy used for crushing a ton of ore. A more sustainable approach to assessing the effectiveness of ore grinding is considered in [5]. Here, in addition to power accounting of the mill electric drive, it is proposed to consider indirect energy consumption - for ore extraction, the manufacturing of the ball, etc. In this case, the implementation of these approaches does not reveal anything new in measuring technological parameters by power, their disadvantages remain and do not provide the necessary accuracy. The results of the studies outlined in [6] expand the understanding of the relationship between the power consumed by the ball mill and the technological parameters, but this data cannot be used to improve the mill load control with an assessment of the energy efficiency of ore grinding. In [7], the calculation of the grindability indices is made. These indices allow quantifying the work expended on grinding in ball mills over the area of the newly created surface, considering all size classes. The work establishes the necessary links, but the parameters cannot be controlled automatically. To reduce the influence of various factors on the measurement of the mill filling level and to increase the accuracy of its measurement, a new variable was proposed [8].

Using theoretical calculations and experiments, the relationship between the mill filling level and the angular position of the maximum vibration point on the ball mill housing was established. Vibration signals were measured with an accelerometer mounted on the mill shell to establish a correlation between the mill filling level and the angular position. It has been established that the position of the maximum vibration point on the mill housing changes to a lower angular position when the drum filling level is increased. The new variable proposed in this paper is more stable. However, the filling level of the mill is determined approximately by an indirect method. It was proved in [9] that the total collision energy of a grinding media per unit time significantly increases with optimal parameters of a ball mill operation. However, the energy efficiency of ore breakage cannot be determined due to the influence on the process of the ball load amount in the drum, the characteristics of its size, lining state and other factors. Despite this, the task of the energy efficiency control, improving ore grinding in ball mills of the first stage is relevant and requires further study. It is more efficient to control the energy efficiency of ore grinding in ball mills by direct energy loss during the destruction of material in the drum, i.e., to perform direct measurement of losses, since the strain analytical dependencies of the unloaded and loaded elastic element are obtained upon the ball's impact.

3. Unresolved parts of a common problem

Based on the obtained dependencies, it was concluded that it is possible to directly measure the energy efficiency of ore grinding, directly in a ball mill. This approach provides an objective assessment of the ore grinding state. However, the effect of a possible change in the balls mass on the results of automatic control of the material destruction energy efficiency has not been studied.

4. Aim of the research

The aim of the research is the mathematical modelling of the ore breakage in a ball mill with the accuracy characteristics determination of the grinding energy efficiency automatic control under conditions of a possible change in the ball mass.

5. Method

At the first stages of ore grinding at concentrating plants, the powerful ball mills are used, in which it is expedient to process a specific technological type of ore and maintain the ball load at a given level and characteristic of ore size, which can provide the highest productivity. Such a constant in time ball load is distributed along with the drum by size due to longitudinal segregation. At the end of the feed are the smallest balls, and at the end of the drum - the largest. Their size changes gradually, creating zones along with the drum with approximately the same size of balls. In each of these zones, due to transverse segregation, the largest balls are located closer to the drum axis, and the smallest ones are at the lining. So, the outer layer of balls of a certain zone will be represented by balls of almost the same size due to two types of segregation. The analysis shows that it is convenient to organize the control in the zone where the size of the balls is 50 mm. The diameter of the balls, and accordingly their mass may differ slightly from the average value. The effect of changes in the mass of balls on the results of automatic control of the energy efficiency of ore grinding can be set in accordance with mathematical models

$$x_1 = \frac{mg + \sqrt{mg(mg + 2ch)}}{c}, \quad (1)$$

$$x_2 = \frac{mg + \sqrt{mg(mg + 2ch) - 2ckk_1V_p}}{c}, \quad (2)$$

where x_1, x_2 - are the deformation of the unloaded and loaded elastic element; m - is the weight of the falling ball; g - is the gravitational acceleration; c - stiffness of the elastic element; h - is the height from which the ball falls - the equivalent of its speed; k - is the proportionality coefficient depending on the strength of the ore; k_1 - is the constant characterizing the relationship between the total and the deformed volume of the ore piece; V_P - is the volume of the ore piece.

The dependence determines the separation angle α of the outer layer balls from the lining

$$\alpha = arc\cos\frac{\pi^2 n^2 R}{900g}, \quad (3)$$

where π=3,14; n - is the number of rounds of the ball mill drum per unit time, rpm.; R - is the inner radius of the mill drum.

The speed of falling of the outer layer balls at the point of collision with the lining is equal to

$$\upsilon_p = \upsilon\sqrt{1 + 8\sin^2\alpha}, \quad (4)$$

where $\upsilon = \pi Rn/30$, m/s.

From (3) and (4) it is seen that the speed of falling balls in the mill is a constant value. Therefore, it is possible to use mathematical models and (2) for h=const. The speed of the balls falling does not depend on their mass, and the deformation of the elastic element depends on the mass.

In (1) and (2), the parameters c and h are constant. If the ore pieces are not large enough, and the crushed ore type is specific, then k, k_1 are also constant. Then, at a particular ball mass m, it is possible to determine the volume of the ground ore V_p by the deformation value x_2 of the elastic element, which corresponds to the energy efficiency of its grinding.

Considering that the change in the mass of the balls affects the deformation of the elastic element, we should expect an error of automatic control of the energy efficiency of ore grinding from this factor. The analysis shows that in this case, the compensation of the error mentioned above can be used. If the control is carried out with the use of (2), then at a constant value of the ground ore volume V_p, a change in the mass of the ball m will lead to an error.

If we also use (1), then we will have at our disposal two strain values x_1 and x_2, caused by a specific mass. The difference in these deformations $\Delta x = x_1 - x_2$ to a certain extent compensates for the effect of changes in the mass of the ball, but not quite, since kinetic energy must be spent on the grinding of the ore, which also depends on the mass of the grinding body moving at a constant speed.

Specific dependencies can be obtained in the process of modelling. Let's take h=4,0872 m for the non-weared up lining of a ball mill, k=3.2 kgm/cm^3, k_1=0.3, V_p=1 cm^3, s=35555555.5 N/m. The simulation results are shown in Fig. 1, where the mass of the balls was changed by ±10%. From Fig. 1 it follows that with a constant diameter (mass) of the balls, the measurement of the ore grinding energy efficiency in a ball mill is carried out without error (curve *2*). If the mass of the balls is increased by 10%, then a measurement error occurs, which depends on the mass of the balls, which are used in the measurement (curve *3*). The largest error will appear in the measurement for balls with a diameter of 40 mm. It is impossible to measure such a high error. The error rapidly decreases for d_k=50 mm and more, depending on the growth of the size of the balls. A decrease in the mass of the ball relative to the normative mass (curve

1) also leads to a measurement error with the same features. Hence, it is possible to effectively control the energy-efficient ore grinding by balls with a diameter of 50 mm, in this case, the error is relatively small, and the size of the ore pieces is also small. It is not advisable to use large ball diameters, especially after d_k=60 mm, since large delays occur and too small pieces of ore are found. From the dependencies obtained, it also follows that the error is compensated - with a 10% deviation of the mass, the measurement error is almost half as much, but it is too large. The errors are almost symmetrical in sign. In the beneficiation processes, they should be significantly less, within 5% [10], even in separate processes. Therefore, it is advisable to focus on the total error in measuring the technological parameters of beneficiation, which should not exceed 3.0%, given that this technological parameter is new. Ensuring the required accuracy of information tools in the management of beneficiation processes is mentioned in [11]. Given this requirement, it is necessary to improve the accuracy of measuring the energy efficiency of ore grinding by ball mills.

The obtained measurement errors of this technological parameter are shown in Fig. 1 and correspond to 1 cm^3 of broken ore. Currently, there is a clear tendency to reduce the coarseness in the crushing department and certain unloading of ball mills by reducing the size of the initial ore. Under these conditions, considering the kinetics of grinding, a smaller product will be produced by a ⅓-½ of the ball mill drum length, which will make it possible to grind a much smaller volume of ore in certain sections of the drum V_p. To destroy a specific volume of ore with a ball, it is necessary to expend a certain amount of kinetic energy. The reduced ball loss of kinetic energy will increase the effect of compensation by determining $\Delta x = x_1 - x_2$, which will be aimed at improving the accuracy of measuring the technological parameter. Let's simulate ore grinding under the same conditions, but with reduced volumes of destroyed material V_p to clarify the possibilities of these changes effect compensation in the mass of the balls on the measurement results. We will change the mass of the ball in the same range ±10%. The simulation results are presented in Tab. 1.

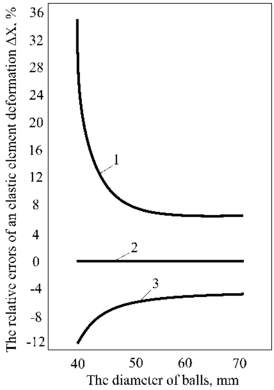

Fig. 1. Change of relative errors of an elastic element deformation with a deviation of balls mass by ±10% in the process of breakage of 1 cm^3 of ore:
1 - reduced; *2* - constant; *3* - increased

From the data of Tab. 1, it can be seen that with an increase in the size of the ball, the measurement errors decrease. They also decrease with a decrease in the volume of broken ore. At the minimum value of the ball mass, an error with a positive sign is formed, and at the maximum value of the mass - with a negative sign. With the same normative values of the ball diameter, the errors are arranged symmetrically and have almost the same values. Despite the reduction of errors in the small volumes of destroyed ore, they remain relatively large and require a reduction.

Tab. 1

Relative deformation errors of the elastic element caused by changes in the mass of the balls (%) at their nominal diameters, mm

The volume of broken ore, V_p, cm^3	Nominal and deviated ball mass	Relative deformation errors of the elastic element caused by changes in the mass of the balls (%) at their nominal diameters, mm			
		40	50	60	70
0.75	min	10.321	6.831	6.108	5.845
	nominal	0	0	0	0
	max	-7.674	-5.654	-5.160	-4.961
0.5	min	8.095	6.258	5.847	5.643
	nominal	0	0	0	0
	max	-7.633	-5.236	-4.972	-4.856
0.25	min	6.217	5.777	5.615	5.410
	nominal	0	0	0	0
	max	-5.234	-4.933	-4.810	-4.696

In a ball mill at the same time, there is a considerable number of balls of different sizes. During operation of the mill, the mass phenomena occur. In particular, it is known that as a result of longitudinal segregation, they are arranged by size when it increases from feed to discharge throat. Segregation is carried out in the cross-sections of the drum when the smallest balls are located at the lining, and larger balls are closer to the drum axis. It is shown that zones of a certain width with the distribution of one-dimensional balls in the outer layer of a ball load are established along with the drum of a ball mill. In the central part of such a zone, there will be, for example, balls with a diameter of 50 mm. However, they cannot exactly correspond to this size. A change in size (mass) within certain limits will lead to errors in measuring the energy efficiency of ore grinding, if the elastic element, as a sensitive device, is installed at a certain point of the drum lining. The interaction of the elastic element with the balls in the process of the mill will be a mass phenomenon.

It is known that mass phenomena have the property of stability, which is the physical content of the "law of large numbers", which in the narrow concept of this word is represented by a number of mathematical theorems. These theorems establish the fact and conditions of convergence in the probability of certain random variables to stable, non-random variables. There is another group of limit theorems, which concerns not the limit values of random variables, but the limit distribution laws. It is also a group of theorems, known as the "central limit theorem".

The general idea of the type of theorem of large numbers law can be formulated as follows. Let a sequence of random variables be given as

$$\xi_1, \xi_2, \dots, \xi_n, \dots . \quad (5)$$

Consider the random variables ξ_n, which are some predetermined symmetric functions of the first n values of a sequence (5)

$$\xi_n = f_n(\xi_1, \xi_2, \dots, \xi_n)$$

If there is such a sequence of constants $a_1, a_2, \dots, a_n, \dots$, which at any $\varepsilon > 0$ are

$$\lim_{n \to \infty} p\left[\left|\xi_n - a_n\right| < \varepsilon\right] = 1 , \quad (6)$$

then the sequence (5) is subjected to the law of large numbers with given functions f_n.

The concept of the large numbers law has a much more specific meaning. Namely, they are limited to the case when fn is the arithmetic mean of the values ($\xi_1,\xi_2,...,\xi_n$). Averaging a sufficiently large number of independent and equally distributed random variables, we obtain a value with a probability arbitrarily close to unity. This value arbitrarily small differs from the general mathematical expectation of values [12].

During the operation of a ball mill, a large number of factors affect the grinding bodies - the interaction of a separate ball with a lining, with ground material, with pulp, with a wide size spectrum of balls, complex ball movements in different layers in a ball mill drum during its rotation, movement along with the drum, in cross-sections of the drum before the lining, etc. Lyapunov showed that if a random variable can be considered as the sum of a large number of small components, then under fairly general conditions, the distribution law of this random variable is close to normal, no matter what the distribution laws of individual components. So, the random variable – the diameter of the balls in a particular standard cross-section of a ball mill drum can be approximately considered as distributed according to normal law.

Let's suppose that when a ball mill is operating, the volume of ore under the balls will not change. However, the deformation of the elastic element will be variable, but considering, for example, all the balls, which are in the normal cross-section of the drum. As a result, we obtain the average value of deformations, corresponding to the mathematical expectation of the ball impact with a diameter of 50 mm, respectively. This value will characterize the volume of ore break under the balls. The mass phenomena here will also respond broken ore volume under the balls, which relates to the average size of the ball of 50 mm. The broken ore volume may occasionally change randomly. However, if we determine the mean deformation value under all balls, which have passed through the elastic element, then with arbitrary accuracy it will characterize the average value of the broken ore volume, i.e., energy efficiency of ore crushing. Let's simulate this process on a simplified mathematical model.

Mathematical modelling will be performed using a ball with a diameter of 50 mm on the same elastic element with c=35555555.5 N/m, ore with k=3.2 kgm/cm^3 and with k=0.3, h=4.0872. The mass of balls in the experiments will be taken as nominal, reduced by 10% and increased by 10%, and the broken ore volumes, respectively – 1.25; 1.0; 0.75; 0.5 and 0.25 cm^3. We will determine the elastic element deformation without ore x_1, with ore x_2 and the difference of these deformations Δx. For each ore volume V_p, we conduct three experiments with balls of different masses, by determining the value x_1,x_2 and x by the averaged data, according to three measurements. The simulation data are presented in Tab. 2.

Tab. 2

Results of mathematical modelling of ore breakage in a ball mill using a simplified mathematical mode

Nominal and deviated ball mass	Deformation of the elastic element without ore x_1, mm	Deformation of the elastic element with ore x_2, mm	Average deformation of an elastic element without ore x_{1c}, mm	Average deformation of an elastic element with ore x_{2c}, mm	The difference of deformations of the elastic element $\Delta x = x_1 - x_2$, mm	The volume of broken ore V_p, cm^3
min	1.01758	0.61090				
nominal	1.07263	0.69916	1.07173	0.69560	0.37620	1.25
max	1.12499	0.77674				
min	1.01758	0.71109				
nominal	1.07263	0.78822	1.07173	0.78570	0.28605	1.0
max	1.12499	0.85778				
min	1.01758	0.79882				
nominal	1.07263	0.86786	1.07173	0.86616	0.20559	0.75
max	1.12499	0.93180				
min	1.01758	0.87782				
nominal	1.07263	0.94110	1.07173	0.93976	0.13199	0.5
max	1.12499	1.00035				
min	1.01758	0.95027				
nominal	1.07263	1.00900	1.07173	1.00792	0.06383	0.25
max	1.12499	1.06450				

The data of the three measurements in Tab. 2 for each value of V_p completely characterize the array of measurements, also considering the intermediate values of the balls mass, since the average value, which corresponding to the nominal one is stable. That is, if all the balls with a variable mass, which are contained in the normal cross-section of the drum, a constant volume of broken ore, for example, V_p=1.25 cm^3, then the mean deformation value x_{2c}, determined by the measured x_2, completely characterizes the volume of the broken material V_p=1.25 cm^3. In a simplified mathematical model, x_{2c} was determined by three values - two limit and one average. The value of the elastic element deformation x_2 at the nominal mass is true. As the analysis shows, deviations of values x_{2c} from x_2 at a nominal mass of the ball is insignificant and depends on the value of the broken ore volume. The values x_{2c} are less then x_2 with the nominal ball mass and change accordingly - 0.51% (with V_p=1.25 cm^3); 0.32% (V_p=1.0 cm^3); 0.20% (V_p=0.75 cm^3); 0.14% (V_p=0.5 cm^3); 0.11% (V_p=0.25 cm^3).

Average deformation of the unloaded elastic element x_{1c} less than x_1 by a constant value - 0.084% with the nominal ball mass. Since the real and experimentally determined values of the elastic element deformation almost coincide, according to Tab. 2, it is possible to build the dependency curves of x_2 (designate as x) and Δx on the value of the broken ore volume, which are presented in Fig. 2. From Fig. 2, it is clear that the curves are non-linear.

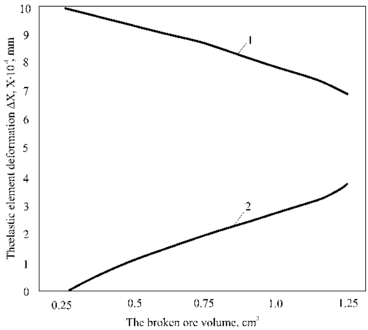

Fig. 2. The dependence of elastic element deformations on the broken ore volume: *1* - deformation of the elastic element with ore; *2* - the deformation difference of the loaded and unloaded elastic element

The graphs obtained in Fig. 2, allowing continue the mathematical modelling of the ore destruction process in a ball mill. If for a specific time the balls in the mill broke all the ore volumes, which are shown in the Tab. 2 and all the deformations were determined, then the mean x_{2c}, which corresponds to all measurements can be determined by data x_{2c} from the table. It is equal to 0.859028, which corresponds to the mean value of the broken volume of 0.75 cm^3. According to curve 1 in Fig. 2 by ordinate 0.859, we determine the value of the broken ore volume of 0.75 cm^3. Thus, for example, according to curve 1 in Fig. 2, it is possible to determine the volume of the broken solid by the measured average strain value of the loaded elastic element, which corresponds to the energy efficiency of ore breakage.

By fitting curve 1 in Fig. 2, the equation for the dependence of the broken ore volume on the expectation of the deformation of the elastic element x_{2c} is obtained

$$V_p = -1{,}9118 x_{2c}^2 + 0{,}0546 x_{2c} + 2{,}1371 \cdot \qquad (7)$$

This equation allows us to find the average value of the broken ore volume, which characterizes the energy efficiency of its grinding in a ball mill for a certain mathematical expectation of the loaded elastic element deformations.

Let's simulate the process of controlling the energy efficiency of ore breakage in a ball mill in accordance with (7). We will use the data of complete ore grinding cycles for volumes of 1.25; 1.0; 0.75; 0.5 and 0,25 cm^3 described in Table 2. Since the volumes of 0.5 and 0.25 cm^3 are rather small, and the ball mill should work at the overload limit, the cycles for 1.25; 1.0; 0.75 cm^3 most fully characterize the work of the technological unit. In the process of modelling, we use various combinations of ore grinding cycles. The simulation results are listed in Tab. 3.

Tab. 3

The mathematical modelling results of the energy-efficient control of ore grinding in a ball mill by a combination of different grinding cycles

Combination of ore crushing cycles with specific volumes V_p, cm^3	Average deformation of an elastic element with ore x_{2c}, mm	The reference value of the destroyed ore volume determined according to Table 2, V_{pe}, cm^3	The volume of broken ore, determined by (7) V_p, cm^3	Relative error of control of broken ore volume δ, %
1.25; 1.0; 0.75	0.782487	1.0	1.009256	0.926
1.0; 0.75; 0.5	0.86387	0.75	0.757546	1.006

0.75; 0.5; 0.25	0.93795	0.5	0.506406	1.280
1.25; 1.25; 1.0; 0.75	0.76076	1.0625	1.072158	0.910
1.25; 1.0; 1.0; 0.75	0.78329	1.0	1.006896	0.690
1.25; 1.0; 0.75; 0.75	0.803405	0.9375	0.946976	1.011

From Table 3 it is clear that when measuring mass and using various combinations of the mass of the ball and broken ore volumes it is possible to determine the averaged value of the broken ore volume, which characterizes the grinding energy efficiency by the measured mean deformations of the loaded elastic element x_{2c} and the resulting analytical dependence.

6. Results and discussion

The proposed mathematical models (1) and (2) can be applied in a ball mill for h=const, since the balls movement speed therein unchanged and independent of their mass. As a result of segregation, the balls are located along with the drum of the mill with an increase in their size (mass) from the load to the throat neck. It is advisable to carry out the measurements with a ball diameter of 50 mm, where the delays are small, and the size of ore pieces is significant. The dependence of the loaded elastic element deformation on a variable within certain limits of the mass of the ball leads to measurement error. This error can be offset by using (1) of an unloaded elastic element. At the same time, in the conditions of 10% deviation of the ball mass, the error of measurement is almost twice less, remaining significant, slightly more than 5%. Reducing the volume of material, which is broken by the falling ball contributes to increasing the direct measurement accuracy of the ore crushing energy efficiency. However, the error practically does not exceed 5%, and this leads to the delay increase. In terms of mass phenomena, a more effective approach may be the interaction of the elastic element with the falling balls. The random values here will be a change in the mass of the balls within certain limits and the broken ore volumes. As the analysis showed, the average value of deformations under all the balls, which passed through the elastic element, also characterizes the average value of the broken ore volume, i.e., the energy efficiency of its grinding.

As a result of the mathematical modelling of this process, the dependences of the elastic element deformation, the difference in the deformations of the unloaded and loaded elastic element on the volume of the broken ore are obtained. They are practically consistent with each other. Considering the insignificant final result of compensation and the strong influence of averaging features on it, it is advisable to consider (1) in Figure 2 and its approximation equation (7). Equation (7) according to the mathematical expectations of the loaded elastic element deformations, allows finding the averaged value of the broken ore volume, which characterizes the energy efficiency of its milling in a ball mill. Mathematical modelling has confirmed the validity of such a conclusion for the random arrangement of ore volumes on an elastic element. The average values of the broken ore volumes (see Tab. 3) determined by (7) are slightly higher than the reference ones. The mismatch of values is basically in the range of 0.690-1.011%. A slightly higher deviation value of 1.28% can be disregarded since it refers to the smallest loads, which can hardly occur during ore grinding. Thus, the deviation of the measured averaged volumes of the broken ore from the reference value does not exceed 1%. The resulting accuracy of the average volume determination meets the requirements of the process.

7. Conclusions

For the first time, the approach of the direct control of ore grinding energy efficiency in ball mills by measuring the mathematical expectation of the elastic element deformation, which interacts with balls and ore in the cross-section of the drum, followed by determination of the broken material volume, which characterizes the measured parameter in accordance with the proposed dependency is proposed. The deviation of the measured averaged volumes of the broken ore from the reference value does not exceed 1%, which satisfies the requirements of the technological process. Automatic control of this technological parameter forms the prerequisites for a significant reduction in energy and material overspending in the first stage of ore grinding at the beneficiation plants.

The prospect of further research is the development of tools for direct measurement of the energy efficiency of ore destruction in ball mills of the first grinding stages at the ore-dressing plants.

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