# THEORETICAL BASIS OF METHODS FOR CALCULATING THE DYNAMIC CHARACTERISTICS OF THE BASES AND FOUNDATIONS OF OIL AND GAS COMPLEX EQUIPMENT





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#### Abstract

The monograph presents a comparison of mathematical and numerical calculation methods as the most promising in this field. Calculations by the above methods are proposed to be carried out according to the second limit state, that is, according to deformations, to determine the amplitudes of oscillations and settlements of the soil and structures. The mathematical method of dynamic phenomena was investigated using an idealized mathematical apparatus. The case of foundation oscillations is described if the soil exerts elastic resistance to compression and displacement.

Calculations to determine the dynamic characteristics of foundations are recommended to be carried out in two directions: using mathematical equations and using software complexes.

The use of a mathematical method in dynamic calculations allows to describe mathematically with the use of certain assumptions and to determine the dynamic characteristics of object oscillations, foundation settlement, which can be described using formulas. And the software complex using the numerical method of finite elements makes it possible to obtain a complete description of the field of vibration displacements, vibration accelerations, and vibration velocities. At the same time, graphs of the dependence of dynamic characteristics at any point of the calculation scheme on time and a graphic description of the deformed grid were obtained.

#### 1. Introduction

The development of the economic sphere of Ukraine involves a significant increase in production rates at enterprises of the oil and gas complex. This leads to an increase in the number of machines and equipment that exert a dynamic influence on foundations and structures located near industrial facilities.

Today, the design of the reconstruction of the foundations of various machines in accordance with building regulations is carried out according to the analytical expressions of the amplitude of the foundation vibrations, depending on the nature of the dynamic influence. Thus, it is not always possible to assess the dynamic state of the entire foundation and its foundations. Therefore, it is necessary to pay attention to the application of numerical methods in the calculation and modeling of dynamic processes, taking into account the nonlinear behavior of the soil under complex stress conditions.

At the same time, it remains an important task to improve the methods of calculating the parameters of oscillations and settlements during the reconstruction of the bases and foundations of machines. Calculation of these characteristics is carried out on the basis of the simplest soil models, which are unable to sufficiently take into account a number of dynamic factors and soil conditions of the research site. Such methods of determining the amplitudes of oscillations and subsidence do not take into account cracks and damage to the foundations of the equipment, changes in the properties of the soil around the foundation that occurred during operation, physical and mechanical characteristics of the soil of the bearing layer, as well as the underlying layers, which can only be taken into account by numerical methods even at the stage of reconstruction . Therefore, the study of foundations at the stage of reconstruction design, taking into account such features of the "foundation - foundation equipment" system, is quite relevant.

## 2. Basic investigation

# 2.1. Basic methods of dynamic calculations of bases and foundations

Calculations to determine the dynamic characteristics of foundations are carried out in two directions:

Dynamic calculations are performed in accordance with the mechanics section "dynamics" using the dynamic characteristics of materials and soils. The Lagrange method is used to derive the equations of motion of the foundation. This method requires a preliminary determination of the value of the mechanical energy of the system. Which consists of the kinetic energy of vertical, horizontal movement and rotational movement and the potential energy of soil shear deformation in the plane of the foundation sole.

If the foundation is buried in the soil, then the potential energy of deformation of the lateral compression of the soil is determined, the reduction of potential energy as a result of lowering the center of gravity of inertia. As a result of the calculations, we have the differential equations of forced planar harmonic vibrations of the foundation, which can be considered generally known in the case of application of transformations.

1. Dynamic loads are converted into static loads using special approaches, and further calculations are carried out in static mode. When considering the system of forces, use the principle of J. D'Alembert. At the same time, inertial forces  $(U_x, U_z, M)$  are added to all external forces and the system is considered as being in equilibrium.



Fig. 1. Calculation scheme of the method of calculating dynamic characteristics

2. This approach is called the kinetostatics method

Both approaches are fair and give an equally correct solution. Consider the application of the kinetostatics method to the derivation of the equations of motion, as the most widespread approach.

We assume that the center of inertia of the foundation and the machine and the center of gravity of the sole are located on the same vertical lying in the main central plane of the foundation. The external loads of the machine, located in the same plane, are brought to the center of inertia of the oscillating mass. They can be represented in the form of an applied force P at this point and a pair of forces with a moment M. Let's combine the origin of the coordinates of the system with the center of inertia of the mass of the foundation and the machine, when the foundation is stationary.

Under the action of loads P,M, the foundation will make a planar movement (Fig. 1). The movement of all points of the foundation will be determined by the values of three independent parameters: x, z projections of the displacement of the center of gravity of the foundation on the coordinate axes with the angle of rotation of the foundation relative to the y axis, which passes through the center of inertia of the foundation and the machine perpendicular to the plane of oscillation. We project all the forces acting on the foundation at the moment of time t on the x,z axis, adding to them the projections on the same axes of inertial forces.

Under the action of dynamic loads, the base is assumed to be linearly deformed, ideally elastic-viscous and massless (soil inertia is not taken into account). The viscosity of the base is determined by the damping properties of the soil. The properties of the base are determined by the coefficients of elastic uniform compression  $C_z$  for vertical vibrations, elastic uniform displacement  $C_x$  for horizontal vibrations, elastically non-uniform compression  $C_{\varphi}$  and elastically non-uniform displacement  $C_{\psi}$  for rotational vibrations and stiffness coefficients for natural bases under elastic uniform compression  $K_z$ , during elastic non-uniform compression  $K_{\varphi}$ , under elastic nonuniform compression shifts  $K_x$ .

The damping properties of the foundation are determined by the characteristics of relative damping during vertical, horizontal, and rotational oscillations relative to the horizontal and vertical axes, which depend on the average pressure under the foundation sole.

The damping properties of the foundation are determined by the characteristics of the relative damping  $\xi$  during vertical, horizontal, and rotational oscillations relative to the horizontal and vertical axes, which depend on the average pressure under the sole of the foundation.

Thus, differential equations of forced oscillations of a system with one degree of freedom were obtained [1].

Formulas for determining the amplitudes of forced vibrations of foundations are obtained as a result of solving differential equations.

This approach to the calculation of machine foundations has been used in domestic engineering practice for the past 20 years.

The production technology or the operation of the machine may require control of the amount of settlement of the machine foundation.

The settlement of the machine foundation caused by dynamic load is determined by the method described by O.O. Savinov. by the formula

$$S_{d} = \sum_{i=1}^{i=n} h_{i} \frac{\varepsilon_{i} - \varepsilon_{0i}}{1 + \varepsilon_{i}}, \qquad (1)$$

 $\varepsilon_i$  and  $\varepsilon_{0i}$  - porosity coefficients of the natural and compacted layer, respectively;

h - the depth of the soil layer with the same porosity coefficients.

When calculating the dynamic impact on houses when driving piles in accordance with building regulations [2], use the formula for determining the horizontal or vertical components of soil movements (B.B. Golitsyn's formula)

$$A = A_0 (r_0 / r)^{0.5e^{-\delta(r - r_0)}}$$
<sup>(2)</sup>

A and  $A_0$  - the amplitude of the horizontal or vertical component movements of the soil at distances, respectively  $r i r_0$ ;

 $\delta$  - vibration damping coefficient, m-1 according to data [2] according to Table 2 for hard clays and loams, it is 0,03 m<sup>-1</sup>;

According to [2], the damping coefficient of soil vibrations by distance is taken according to table 2 and specified according to measurement data according to regulatory documents.

On the one hand, the advantage of formula (2) is its simplicity, and on the other hand, its flexibility when approximating it using the results of observations of movement amplitudes on different sites, which are composed of different soils.

This flexibility is achieved due to the possibility of entering into it different values of  $A_1$ , including those measured at the distance  $r_1$  and due to the substitution of different values of the damping coefficient depending on the soil conditions of the construction site.

Another technique based on the detection of dependencies between the parameters of soil surface fluctuations and its strength characteristics obtained during static sounding was presented by M. Kalyuzhniuk, V.Rud. For driving piles in sandy and clay soils, the formulas for calculating the vibration parameters differ in the values of the coefficients included in the formula

$$V = k_{\nu \max} q_s (2/r)^{t_{g\nu}}$$
(3)

then the maximum vibration displacement is determined by the formula

$$A = \frac{0.16 \quad k_{v \max} q_s (2/r)^{tgv}}{\omega} \tag{4}$$

 $k_{vmax}$  - coefficient, which depends on the source of vibrations and the type of soil, for piles;

 $q_s$  - resistance to immersion of the probe cone during static probing, MPa;

r - the distance from the pile to the point at which the vibration parameters are determined, m;

 $\omega$  - frequency of soil vibrations, Hz; tgv - degree indicator, which depends on the characteristics of the upper layer of the soil with a thickness of 5-10 m.

The seismic impact of blasting is assessed using vibration velocity and vibration acceleration in comparison with the values of the MSK scale.

There are methods for determining the vibration acceleration of soil particles in the radial direction from the center of the explosion, which are based on the formula of M. Sadovskyi and differ only in coefficients included in the formula determined experimentally. They are not presented in normative literature but are used in engineering calculations.

Vibration acceleration of soil particles in the radial direction based on experimental data according to S. Medvedev [3] should be determined by the formula

$$a = 6, 4 \cdot K_s \omega \sqrt{\frac{g}{\rho b_1 \tau}} \left(\frac{\sqrt[3]{C}}{R}\right)^{1.5}$$
(5)

 $K_{e}$  - empirical coefficient, m;  $\rho$  - soil density, kg/m<sup>3</sup>; g - acceleration of gravity, m/s<sup>2</sup>;  $\tau$  - period of soil oscillation;  $b_1$  – speed of wave propagation, m/s (determined according to regulatory documents).

The maximum vibration acceleration of displacement of soil particles at the point of determination according to V.G. Afoninum is determined by the formula [4]

$$a = 6,4\omega \cdot (2,50 - 1,50)(\frac{\sqrt[3]{C}}{R})^{2.5}$$
(6)

*C* - charge weight, kg; *R* - distance to the place of explosion, m;  $\omega$  - frequency of oscillations, Hz (used for calculation  $\omega = 0.3$  Hz).

The author of the methodology for determining the characteristics of soil oscillations gives formulas for determining the speed, which can be transformed into the given expressions for determining the vibration acceleration of soil oscillations with the help of known formulas.

To assess the intensity of vibrations during explosions, the same scale is used as during earthquakes, according to which a certain amount of vibration acceleration of vibrations corresponds to a certain point of seismicity (according to MSK). The advantages of analytical methods for calculating dynamic characteristics are ease of use, it does not require special skills and abilities from the engineerdesigner.

Disadvantages are the impossibility to take into account a sufficient number of factors affecting the dynamic process by mathematical dependence. Because in order to be able to describe the phenomenon with the help of a mathematical apparatus, a certain number of assumptions are accepted.

## 2.2 Dynamic calculation using the finite element method

In addition to analytical methods, numerical methods for calculating dynamic characteristics, namely the finite element method, have recently been widely used to solve dynamics problems.

The choice of calculation method and specific model depends on the problem to be solved. Most of the developed software complexes are tested for certain types of foundations and soil conditions and do not allow calculations for a wide range of foundations, soil foundations, loading and deformation conditions.

Each software complex is based on a specific model of the soil environment. Evaluation and selection of a software tool must be carried out with the involvement of certain requirements that are established to solve a specific problem. Currently, many software complexes have been created for dynamic calculations of structures, bases and foundations of buildings and structures.

Each of the programs has both its advantages and disadvantages from the point of view of solving a specific problem. For dynamic calculations, a flat and spatial resolution of the problem is adopted. The most accurate results of calculations are obtained in the case of a spatial formulation of problem solutions, but such solutions are often quite complex compared to a flat formulation. In turn, the flat problem is more universal and simple in the implementation of complex soil models, in addition, it does not require significant computing resources. If you set adequate initial data, taking into account all the features of bringing the work of the soil massif and the structure or building to a flat scheme, you can get sufficiently reliable results.

The following are among the multi-purpose software complexes: "Lira", "MSC.Nastran", "APM Dynamics", "Dinamika - 3", "NONSAP", "Selena", "SCAD", "ABAQUS", "Zenith-95". These software complexes allow to perform both static and dynamic calculations of structures.

Software complexes suitable only for dynamic calculation of any structures are "ANSYS LS-DYNA", "LS-DYNA", "MSC/DYTRAN", "T-Flex/Dynamics".

Software complexes of a narrow profile are designed for dynamic calculations of certain types of bases and foundations. Such programs include "Dynardo", which can be used to perform a dynamic analysis of foundations (piles and slabs) compatible with the soil on the effect of wind load. "Dynamics" is a software complex for calculating port hydraulic structures, machine foundations.

The market of programs for engineering calculations offers enough of both domestic and foreign developments, which allow to perform calculations of load-bearing structures and their aboveground part with a high degree of reliability. But a less researched field is related to geotechnical engineering calculations, which are based on soil modeling processes, interactions between structures and soil. There are few high-quality and convenient programs for professionals in this field ("CONCORD", "PLAXIS"). "PLAXIS" is a universal program that is suitable for dynamic calculation of most bases and foundations, which is well adapted for calculations of this type of problems. This software complex is based on the use of the finite element method in solving dynamic problems.

The main idea of the finite element method is that any continuous quantity (vibration displacement, vibration velocity, vibration acceleration) is approximated by a discrete model, the construction of which is performed on a set of piecewise continuous functions. The essence of the finite element method is that the structure being calculated is considered to be composed of a finite number of individual elements of a simple geometric shape, tightly adjacent to each other and hinged together at the vertices of these elements. The shape and dimensions of the structure remain unchanged. The shape of the elements can be different and depends on the shape of the structure under consideration. For a flat structure, the triangular shape of the end elements is most suitable. The solid environment of the calculated structure, after dividing it into finite elements, does not lose its basic quality - it remains solid, consisting of separate twodimensional elements.

I will consider the main points of implementation of the method of finite elements in the software complex "PLAXIS" [5].

The static equilibrium of a solid body can be formulated as

$$L^{\prime}\sigma + p = 0. \tag{7}$$

In addition to the equilibrium equation, the kinematic relationship can be formulated as

$$\varepsilon = Lu \tag{8}$$

This equation goes to the 6 stress components and is collected in the vector  $\varepsilon$  as the spatial derivatives of the three displacement components are collected in the vector u using the available differential operator L. The relationship between the equations is given by the soil model. General relation

$$\dot{\sigma} = M\dot{\varepsilon} \tag{9}$$

The equilibrium equation is restated in a simplified form according to the Galerkin change principle.

$$\int \delta u^{T} (L^{T} \sigma + p) dV = 0 \,. \tag{10}$$

In this formulation,  $\delta u$  represents a kinematically admissible change in displacements. Application of Green's theorem for partial integration over the first value in equation (7) leads to

$$\int \delta \varepsilon^{T} \sigma dV = \int \delta u^{T} p dV + \int \delta u^{T} t dS \cdot$$
<sup>(11)</sup>

This is a boundary integral in which boundary displacements appear. 3 components of the limit displacement are collected in the vector t.

Equation (10) is referred to the equation of virtual work. The development of the stress state  $\sigma$  can be regarded as a constantly growing process

$$\sigma^{i} = \sigma^{i-1} + \Delta\sigma, \ \Delta\sigma = \int \dot{\sigma} dt \,. \tag{12}$$

 $\sigma^i$  represents the actual stress state;

 $\sigma^{i-1}$  represents the previous state of stress;

 $\Delta\sigma$  - is the norm of the integrated stress over a small increment time.

If equation (11) is considered for the corresponding distance and *i*.

$$\int \delta \varepsilon^T \Delta \sigma dV = \int \delta u^T p^i dV + \int \delta u^T t^i dS - \int \delta \varepsilon^T \sigma^{i-1} dV$$
(13)

It can be noted that the appearance of quantities in equations (7) to (13) is a function of position in three-dimensional space.

According to the finite element method, each element consists of a node number. Each node has a number of degrees of freedom, which is described by discrete values that are known within the limits of the desired value. Degrees of freedom are assigned to the displacement components. In the move field element u

$$u = Nv \tag{14}$$

The interpolation of functions in the matrix N is often denoted by shift functions. Replacing equation (14) in the kinematic relation is given as

$$\varepsilon = LN\nu = B\nu \tag{15}$$

In this regard, B is the interpolated stress matrix, which contains the spatial derivatives of the interpolated functions. Equation (13) can be reformulated in the described form as

 $\int (B\delta v)^T \Delta \sigma dV = \int (N\delta v)^T p^i dV + \int (N\delta v)^T t^i dS - \int (B\delta v)^T \sigma^{i-1} dV (16)$ Discrete displacements can be placed outside the integration.

$$\delta v^{T} \int B^{T} \Delta \sigma dV = \delta v^{T} \int N^{T} p^{i} dV + \delta v^{T} \int N^{T} t^{i} dS - \delta v^{T} \int B^{T} \sigma^{i-1} dV$$
(17)

Якщо рівняння (2.20) містить будь-які зміни приростів  $\delta v^{T}$ , рівняння може бути записане як:

$$\int B^{T} \Delta \sigma dV = \int N^{T} p^{i} dV + \int N^{T} t^{i} dS - \int B^{T} \sigma^{i-1} dV$$
<sup>(18)</sup>

Equation (18) is the equilibrium condition in discretized form, the first quantity on the right together with the second quantity represents the flow of the external force vector, and the last quantity represents the internal reaction vector. The difference between the external force vector and the internal reaction vector can be balanced by the stress increment  $\Delta \sigma$ . The relationship between the increase in tension and the increase in compression is usually not linear. As a result of the increase in compression, they may not be directly calculated and generally require repeating operations to satisfy the equilibrium condition (18) for all points of the material.

Consider the procedure of implicit integration of differential plastic models. The stress increments  $\Delta\sigma$  re obtained by integrating the normal stress according to equation (12). For differential plastic models, stress increments can generally be written as

$$\Delta \sigma = D^{e} (\Delta \varepsilon - \Delta \varepsilon^{p}) \tag{19}$$

 $D^e$  is the elasticity matrix of the material for the stress increment flow. The compression increment  $\Delta \varepsilon$  is obtained from the displacement increment  $\Delta v$  using the interpolation of the compression matrix similar to equation (14) For the elastic behavior of the material, the plastic compression displacement  $\Delta \varepsilon^p = 0$ . For the plastic behavior of the material, the increase in compressive stress can be written accordingly

$$\Delta \varepsilon^{p} = \Delta \lambda \left[ , (1 - \omega) \left( \frac{\partial g}{\partial \sigma} \right)^{i-1} + \varpi \left( \frac{\partial g}{\partial \sigma} \right)^{i} \right]$$
(20)

In this equation,  $\Delta \lambda$  there is a displacement of the plastic multiplier and  $\omega$  is a parameter that determines the type and time of integration.

For integration  $\omega=1$  it is called explicit and for integration it is called implicit. Vermeer showed that using implicit integration ( $\omega=1$ ) has some advantages.

In addition, it can be proven by implicit integration, under some conditions, leads to symmetry and positive differentiation of the matrix  $\partial e \partial \sigma$  with repeated procedures has a positive effect. Because these main advantages are limited to implicit integration, they are not true for other types of time integration.

However,  $\omega=1$  equation (20) is simplified to

$$\Delta \varepsilon^{p} = \Delta \lambda (\frac{\partial g}{\partial \sigma})^{i}$$
(21)

Substituting equation (21) into equation (19) and successively into the equation

$$\sigma^{i} = \sigma^{tr} - \Delta \lambda D^{e} \left(\frac{\partial g}{\partial \sigma}\right)^{i} c \sigma^{tr} = \sigma^{i+1} + D^{e} \Delta \varepsilon$$
(23)

In this regard  $\sigma^{ir}$  is the auxiliary stress vector related to the elastic stresses or to the test stresses, which is the new state of stress when we consider the fully elastic behavior of the material. Increase in plastic product  $\Delta\lambda$ , used in equation (23) can be solved from the condition that the new stress state satisfies the initial condition

$$f(\sigma^i) = 0 \tag{24}$$

For perfectly plastic and linearly fixed models, the increment of the plastic multiplier can be written as follows

$$\Delta \lambda = f(\sigma^{\prime r}) / (d+h)$$
<sup>(23)</sup>

where

$$d = \left(\frac{\partial f}{\partial \sigma}\right)^{\sigma^{I'}} D^{e} \left(\frac{\partial g}{\partial \sigma}\right)^{i}$$
(26)

the symbol h stands for the steady parameter, which is 0 for perfectly plastic models and constant for linear steady models.

Replacing the ratio between stress increments and stresscompression increments  $\Delta \sigma = M \Delta \varepsilon$  in the equilibrium equation leads to

$$K^i \Delta v^i = f^i_{ex} - f^{i-1}_{in} \tag{27}$$

In this equation K - stiffness matrix,  $\Delta v$  - displacement increment vector,  $f_{ex}$  - external force vector i  $f_{in}$  - internal reaction vector. The superscript refers to the step number. The relationship between stress increment and compressive stress increment is not linear. The stiffness matrix cannot be formulated precisely in advance.

However, the general iterative procedure requires both equilibrium conditions to be satisfied. The general iterative process can be described as

$$K^{j}\delta\nu^{j} = f_{ex}^{i} - f_{in}^{j-1}$$
<sup>(28)</sup>

The upper symbol *j* refers to the integration number,  $\delta v$  - a vector containing displacements that gradually increase, that contribute to displacement increments with a step and:

$$\Delta \nu^{i} = \sum_{j=1}^{n} \delta \nu^{j} \tag{29}$$

n - the number of iterations in step i, the stiffness matrix K, used in equation (28) represents the behavior of the material in an approximate form. In its simplest form, K is a linear elastic solution. In this case, the stiffness matrix can be formulated

$$K = \int B^T D^e B dV \tag{30}$$

 $D^e$  - the elastic matrix of the material according to Hooke's law and B – compression stress interpolation matrix. Using an elasticstiff matrix is a real iterative procedure until the stiffness of the material increases, even when using incoherent plastic models. To improve practical application, we have an automatic step size procedure. For material models with linear behavior in the elastic region, we use the Mohr-Coulomb model.

The processes of movement are described by the interpolation function of the elements. Within the boundaries of the displacement field element  $u = (u_x, u_y)^T$  vector obtained from the discrete central value  $v = (v_1v_2...v_n)^T$  using the interpolation function is assembled into a matrix N

$$u = Nv \tag{31}$$

However, the interpolation function N is used to interpolate the values inside the considered element on the known values in the center. Linear elements are based on geogrid elements, planar elements and distributed loads. When the position  $\xi$  of a point (usually a stress point or an integration point) is known, one can write for the displacement component u.

$$u(\xi) = \sum_{i=1}^{u} N_i(\xi) v_i$$
 (32)

 $v_l$  - central values;

 $N_i(\xi)$  - the value of the displacement functions of the node and at the position  $\xi$ ;

 $u(\xi)$  - the resulting value of u at the position  $\xi$ ;

*n* - number of element nodes.

In the program, we have movement functions for 15 nodal and 6 nodal triangular elements [5].

The basic equation of volume movement depending on time under the influence of dynamic load

$$M\ddot{u} + C\dot{u} + Ku = F , \qquad (33)$$

M - mass matrix; u - displacement vector; C - damping matrix; K - stiffness matrix; F - force vector.

Displacement u, velocity  $\dot{u}$ , acceleration can change over time.

The value  $K \cdot u = F$  is rewritten as for the static deformation calculation. Matrix K contains material stiffness properties, vector Fcontains load components. Each element is characterized by a stiffness matrix that establishes a relationship between nodal forces and nodal displacements of the element depending on the coordinates of its nodes and the elastic properties of the material. Also, the entire design being calculated is characterized by a generalized stiffness matrix of the system, which consists of the stiffness matrices of all final elements included in its composition. We impose boundary conditions and nodal forces on this matrix. After integrating the stiffness matrix, we obtain the components of movements in all nodes of the system. In relation to stresses in the soil, the described theory is based on the linear elastic behavior of the soil. In the case of wet soil, a larger part of the soil water stiffness is added to the stiffness matrix, as in the case of static calculation. The mass matrix takes into account the mass of soil, the mass of soil water, and the mass of structures. The mass is presented as a matrix of individual load elements (soil, water, structures).

The matrix C reflects the damping of vibrations in the material. To determine the attenuation matrix, additional parameters are required that are difficult to determine from laboratory tests. Therefore, the matrix C is often described as a combination of the mass matrix and the stiffness matrix (Rayleigh damping)

$$C = \alpha_R M + \beta_R K \,. \tag{34}$$

The boundaries of the attenuation matrix are determined by entering the Rayleigh coefficients  $\alpha_R$  i  $\beta_R$  Parameter  $\alpha_R$  determines the effect of mass on the damping of the system. When the proportion of the mass matrix M in the damping matrix C increases (for example  $\alpha_R=10^{-2}$  and  $\beta_R=10^{-3}$ ) damping of lower vibration frequencies occurs, and when the share of the stiffness matrix K in the damping matrix C increases (for example  $\alpha_R=10^{-3}$  i  $\beta_R=10^{-2}$ ) higher frequencies of oscillations are attenuated. With the standard installation of the program, the attenuation of the Relay is not taken into account  $\alpha_R=\beta_R=0$ .

При динамічних розрахунках форма інтегрування по часу визначається фактором стабільності і точності процесу розрахунку. В схемі розрахунку використовується метод неявного інтегрування по часу. В цьому методі переміщення і швидкість для точки в часі  $t+\Delta t$  записуються відповідно:

In dynamic calculations, the form of integration over time is determined by the factor of stability and accuracy of the calculation process. The calculation scheme uses the method of implicit integration over time. In this method, displacement and velocity for a point in time  $t+\Delta t$  are written respectively

$$\begin{aligned} \ddot{u}^{t+\Delta t} &= c_0 \Delta u - c_2 \dot{u}^t - c_3 \ddot{u}^t, \\ \dot{u}^{t+\Delta t} &= c_1 \Delta u - c_4 \dot{u}^t - c_5 \ddot{u}^t, \\ u^{t+\Delta t} &= u^t + \Delta u. \end{aligned}$$
(35)

 $\Delta t$  – time step Coefficients  $c_0...c_5$  can be expressed in a time step and in the integration of parameters  $\alpha \ i \ \beta$ . Coefficients  $\alpha \ i \ \beta$  are determined by the numerical precision of the integration time and are not equal  $\alpha \ i \ \beta$  for Rayleigh damping. In order to obtain a stable result, the following condition must be applied:  $\beta \ge 0.5$ ;  $\alpha \le 0.25(0.5+\beta)^2$ . Standard values of coefficients can be used in many calculations  $\alpha = 0.25$ ,  $\beta = 0.5$ .

With this method of movement, the speed and acceleration at the end of the time step are expressed through the values at the beginning of the time step and the increment of the movement. With implicit integration, equation (33) must contain the time at the end of the step  $t+\Delta t$ 

$$M\ddot{u}^{t+\Delta t} + C\dot{u}^{t+\Delta t} + Ku^{t+\Delta t} = F^{t+\Delta t}, \qquad (36)$$

This equation is combined with expression (36) for displacements, velocity and acceleration at the end of the time step  $(c_0M + c_1C + K)\Delta u = F_{ext}^{t+\Delta t} + M(c_2\dot{u}^t + c_3\ddot{u}^t) + C(c_4\dot{u}^t + c_5\ddot{u}^t) - F_{int}^t$ , (37)

In this form, the system of equations for dynamic analyzes is also used for static calculations. The difference will be in the stiffness matrix, which contains additional values for mass and damping, and the values of c. The right side of the equation contains additional values for determining the speed, acceleration at the beginning of the time step (time  $\Delta t$ ).

Despite the implicit integration, the time step used in the calculations is limited. If the time step is very large, then as a result of calculations we will have deviations and the result will lose accuracy. The critical time step depends on the maximum frequency of the model and the accuracy of the connection of the finite elements. For a single element we will have

$$\Delta t_{crit} = \frac{B}{\alpha \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}} \sqrt{1 + \frac{B^4}{4S^2} - \frac{B^4}{4S} [1 + \frac{1-2\nu}{4} \frac{2S}{B^2}]}, \quad (39)$$

The value of the first root is the speed of the compression wave. The factor  $\alpha$  depends on the type of element.

For a 6-node element  $\alpha = 1/(6\sqrt{c_6})$ ,  $\exists c_6 \approx 5,1282$ , for a 15-node element  $\alpha = 1/\sqrt{19c_{15}}$ ,  $\exists c_{15} \approx 4,9479$ .

 $\Delta t$  Another significant factor is Poisson's ratio, the average number of the length of the element *B*. The surface *S* is used to describe the average number of the length of the element. In the final model of the element, the critical time step is equal to the minimum value  $\Delta t$  for all elements according to (39). The chosen time step is so small that the wave in a unit step moves a smaller distance than a unit element.

In the case of static strain analyses, the described limit displacements are represented at the boundaries of the element model. For dynamic calculations, the boundaries will be further away than for calculations of static deformations, because rapid dispersion of oscillations occurs. When the boundaries are very close to the dynamically loaded structure, the oscillations quickly reach them and the oscillations are reflected and propagated in the opposite direction. This leads to distortion of calculation results. However, placing the boundaries away from the object requires many additional elements and therefore a lot of additional memory and time.

To prevent these negative consequences, absorption limits are applied. Absorption limits were used in the calculation of dynamic characteristics and subsidence. The components of normal stress and shear stress are absorbed in the direction of the coordinate axes of the calculation scheme. They are described by certain expressions consisting of the product of dynamic characteristics, soil density, transverse and longitudinal wave velocities and absorption coefficients. That is, longitudinal waves are completely attenuated, the attenuation coefficient  $C_1$ =1, and the transverse ones are attenuated only 25%,  $C_2$ =0,25.

These methods have their advantages and disadvantages, and relate to specific research problems in this field. When choosing absorption boundaries, a humidifier was used instead of using fixed boundaries in the main direction. The dampener ensures that the voltage increase at the boundary is absorbed without impact.

With the help of "PLAXIS" software complexes, the effects of soil vibrations and structures can be fully analyzed on the basis of solving the axes of a symmetric and planar problem. They include the final set of a package of modules for analyzes of deformations and stability of geotechnical projects. The dynamic model includes the inertia of the base layer and the load action time. Oscillations weaken with increasing distance from the source of oscillation due to effects similar to geometric damping. When performing calculations, "PLAXIS" can be started in the mode of automatic selection of the value step and the time step. This allows you to avoid choosing the appropriate load increment for plasticity calculations, which guarantees the efficiency and accuracy of the calculation process.

# **2.3 Required output data for dynamic calculations**

1. Data on structures and loads.

The composition of the initial data for the design of the foundations of machines with dynamic loads according to [7] clause 1.1 should include: technical characteristics of the machine (name, type, number of revolutions per minute, total mass, mass of moving

parts, speed of falling parts); data on the value, place of application and directions of action of static and dynamic loads in normal operation, dimensions of load transfer platforms, information on the presence of factory vibration isolation in machines with an indication of dynamic loads transmitted to the foundations taking into account this vibration isolation; data on the limit values of deformations of foundations and their bases (settlement, roll, deflection of the foundation and its elements, amplitude of oscillations), if such limitations are caused by the conditions of production technology, machine operation; data on the conditions of placing the machine (equipment) on the foundations: separate foundations for each machine (aggregate) or their group installation on a common foundation, data on the characteristics of the equipment support plates, data on the type of their connection to the foundation, etc.

When studying soil vibrations due to pile driving, the initial data should be: data on the geometric dimensions of the piles, on the method of their immersion, technical characteristics of the equipment with the help of which the pile immersion process is carried out.

When assessing the impact of blasting, it is necessary to have data on the specifics of blasting and explosives.

2. Data about surrounding objects.

When designing, it is necessary to pay attention to nearby structures and equipment when designing foundations for machines, and buildings and structures when conducting pile-dive and blasting works.

When designing, it is necessary to take into account the presence of high-precision and vibration-sensitive equipment, the condition of buildings and structures located nearby, their structural features, and depending on this, draw conclusions about the permissible amplitude of oscillations.

3. Data on the engineering and geological conditions of the construction site and the physical and mechanical properties of the foundation soils.

For the calculation of foundations according to building regulations, the main characteristic of the soil used in the calculations is the modulus of elasticity of the soil, which is used to calculate the coefficients that take into account the properties of the soil. When calculating the vibration impact from the sinking of piles and from blasting works using the analytical method, engineering geological conditions are taken into account with coefficients that are determined depending on the soil category. Soil density is taken into account.

When using numerical methods of entering the geometry of the soil layers, structure, loads and boundary conditions, it is based on CAD drawing procedures, which provide a detailed and accurate simulation of the real situation. The engineering and geological conditions of the site are taken into account on the basis of such physical and mechanical characteristics of the soil as: soil specific gravity, adhesion, angle of internal friction, modulus of deformation, Poisson's ratio. To enter the geometry in "PLAXIS", such elements as a beam, a hinge, contact surfaces, anchors, geotextiles (geogrids), tunnels, boundary conditions, loads are presented. When performing geotechnical calculations, it is necessary to have basic soil models to simulate nonlinear and non-stationary behavior of soils. At the same time, it is necessary to take into account the soil substrate itself, hydrostatic and non-hydrostatic pore pressure in it. Thus, the main emphasis is placed on the interaction of the soil and those structures that can be erected on this site. In the absence of any category of initial data, it is impossible to carry out further calculations using analytical or numerical methods.

# 2.4 Peculiarities of drawing up calculation schemes

For calculating the dynamic characteristics and settlement of the foundation, the compiled calculation scheme is important. Calculation schemes compiled for calculation have their own characteristics.

When calculating the dynamic characteristics of the foundation for the equipment using the analytical method, the following is accepted

1. The machine-foundation system has 6 degrees of freedom: 3 displacements and 3 rotations, which can be replaced by research with an equivalent, specially selected system with one degree of freedom (for vertical or horizontal oscillations) or with two degrees of freedom (vertical or horizontal oscillations taking into account rotational or rotational components of vibrations).

1. The coordinate system is chosen to pass through the common center of gravity of the machine and the foundation. All acting

forces, moments, displacements act on some point of the foundation relative to these axes. It is assumed that the system has 2 vertical planes of symmetry and the horizontal axes lie in those planes. These axes are the main axes of the body, relative to which the inertia forces are zero.



Fig 2. Calculation scheme for calculation of soil vibrations during pile immersion by impact method

2. In the case of elastic non-uniform compression (rotation of the foundation sole relative to the horizontal axis passing through the center of gravity of the foundation sole perpendicular to the plane of oscillations), it is permissible to assume that the plane of oscillations is parallel to the line of action of the acting force or the plane of action of the disturbing moment.

3. However, there are no instructions in the construction regulations to take into account the deepening and backfilling, but the instructions [8] state that for a more accurate assessment of the stiffness coefficients of the base, the influence of the lateral backfilling of the foundation on the increase of these coefficients should be taken into account, conducting special studies for this purpose.

When calculating soil vibrations from driving piles, when choosing a mathematical model of a pile as a source of vibrations, it is considered as an axis of a symmetrical source. Wavelengths propagated in the soil during pile driving are much larger than its transverse dimensions, so although the pile is a volume source, it can be replaced by the line of action of forces. Plastic deformations in the region in the immediate vicinity of the pile are neglected and the problem is treated as elastic. That is, we have a perfectly elastic homogeneous half-space and a linear axis symmetric pulse source of oscillations in the half-space . We have a uniform nature of load distribution along the line of action of the force (Fig. 2.4). 1st, 2nd, and 3rd regions in the half-space, conventionally separated by planes  $z=h_1$ ,  $z=h_2$ .

Seismic phenomena during an explosion are a small model of a natural earthquake. The source of seismic energy during an explosion is the explosive charge. That is, the explosion phenomenon can be represented as the loading of an elastic half-space by a concentrated force, which reveals the main regularities of the formation and propagation of seismic vibrations (Fig. 3).



**Fig 3.** Calculation scheme for calculating soil vibrations from explosions during rock development

The calculation scheme used in calculating the dynamic characteristics of equipment foundations, soil vibrations and subsidence due to dynamic influence using the finite element method has the following features:

1. When drawing up the calculation scheme, ties and absorption boundaries are superimposed on the side and bottom boundaries. Elms prohibit the movement of the side borders along the abscissa axis, and the lower border along the ordinate and abscissa axes.

2. Dynamic studies in "PLAXIS" software complexes can be divided into analyzes of two types of problems:

- problems with a single source of oscillations (oscillations of machine foundations, soil oscillations during pile driving);

- problems of earthquakes. Earthquakes can be modeled by describing movements with a dominant horizontal component.

During an earthquake, the dynamic load acts on the entire plane of the bottom of the model.

3. From the created geometric model, the program automatically generates an unstructured ordinary mesh with the possibility of global and local changes in its density. The use of high-order elements in the model is useful for the uniform distribution of stresses in the soil and the accurate prediction of unacceptable loads. The user is given a choice between 6-node and 15-node elements, which can be successfully used in calculations. 4. The program takes into account advanced models in geotechnics for nonlinear modeling of soil behavior that does not depend on time.

5. To perform calculations, in addition to establishing the basic physical and mechanical characteristics of soil and material (modulus of deformation, specific adhesion, angle of internal friction, specific weight, Poisson's ratio), it is possible to set properties from the point of view of permeability, taking into account filtration. It is possible to use dried and not dried soil in calculations. These conditions are related to the ability of pore water to move between soil particles, which can lead to a change in volume and is accompanied by an excess of additional pore water pressure. When using the Mohr-Coulomb model, it is necessary to enter the values of physical and mechanical characteristics of dry soil during calculations.

Usually, the calculation schemes made by analytical and numerical methods are quite different, but they have common assumptions, for example, in the geotechnical analysis of a certain structure or building (foundation for equipment, pile), they are taken as made of linear-elastic, rigid material, and such that there is no porosity.

## 2.5 Soil base models used in calculations

Conducting a dynamic calculation using analytical and numerical methods is based on the choice of a soil base model, which in a certain way depends on the accuracy of the obtained result.

In the dynamic calculation of foundations under machines using the analytical method, the base is considered as an elastic-viscous linearly deformed medium. The properties of the environment in the calculation are determined by the coefficients of elastic uniform and non-uniform compression, elastic uniform and non-uniform shear and coefficients characterizing damping.

When calculating soil vibrations from pile driving and from explosions, an elastic half-space is used as a soil model. To simplify the solution of the problem, it is assumed that the material of the half-space is perfectly elastic, homogeneous and isotropic, and the relationship between deformations and stresses is linear. In general, all assumptions, with the exception of soil homogeneity, are sufficiently consistent with experimental data during pile driving. Indeed, as a result of the small amplitudes of displacements that occur when driving piles in the first approximation, it can be assumed that the absorption of vibration energy by the soil does not occur. Soil is a perfectly elastic material. For the same reason, the geometric and physical non-linearity of the dependence of soil deformation on stresses can be disregarded. The assumption of soil homogeneity made to explain the general patterns of wave propagation in soils. But the further away from the place of the explosion, the more significant is the influence of the environment and its composition. This is explained by the fact that the medium (soil) differs significantly in its properties from the elastic half-space. Recently, there have been attempts by scientists to improve soil models when describing dynamic phenomena with analytical expressions, but no additions to building regulations have been made.

The development of numerical methods makes it possible to bring the model of the soil environment closer to the real one when calculating dynamic phenomena. When modeling the dynamic processes of foundations and foundations, a static calculation was first performed. After the static calculation, the dynamic behavior of foundations and foundations was modeled. During the static calculation, discrete models were used to simulate the non-linear, rheological behavior of soils. The Mohr-Coulomb nonlinear model is based on soil parameters that are known in most cases.

This phenomenological model of soil deformation involves taking into account physical and geometric nonlinearity, dilatancy, compaction in the process of deformation, pore and hydrostatic pressure, shrinkage and swelling of the soil.

The Mohr-Coulomb elastic-plastic model is based on 5 input parameters: modulus of elasticity and Poisson's ratio as for elastic soil, angle  $\varphi$  and specific adhesion as for plastic soil and  $\psi$ - angle of dilatancy (spreading). General deformations include linear (elastic) and plastic parts, and the plastic component of deformations occurs after the stress state reaches the limit of proportionality (yield, strength).

The relationship between stresses and strains is bilinear, Fig. 4.

For the limit of proportionality at the point (elementary volume) of the array, for the condition of plane deformation, the Mohr-Coulomb equation is used in the form:

$$\frac{\sigma_1 - \sigma_2}{2} + \frac{(\sigma_1 + \sigma_2)\sin\varphi}{2} - c\cos\varphi = 0$$

Fig 4. Graph of dependence between stresses and deformations of an elastic-plastic medium

Plasticity is associated with the development of irreversible deformations. In order to estimate the amount of plasticity in the calculations, the stretching (stress) function is introduced. The stretch (stress) function

can often be represented as a surface in the principal stress space.



σ

**Fig. 5.** The cone of principal stresses e space of the Mohr-Coulomb model

A perfectly plastic model is based on a model with a constant stretching surface, that is, the stretching surface is completely determined by the model parameters and is not subject to plastic deformations. The Mohr-

Coulomb condition is an application of the law of friction

Coulomb to general stress states. In fact, this condition guarantees that Coulomb's law of friction holds in any plane within the material element.

The complete Mohr-Coulomb stretch condition can be defined by three stretch (stress) functions when the principal stress conditions are formulated

$$f_{1} = \frac{1}{2}(\sigma_{2}' - \sigma_{3}') + \frac{1}{2}(\sigma_{2}' + \sigma_{3}')\sin\varphi - c\cos\varphi \le 0,$$

$$f_{2} = \frac{1}{2}(\sigma_{3}' - \sigma_{1}') + \frac{1}{2}(\sigma_{3}' + \sigma_{1}')\sin\varphi - c\cos\varphi \le 0,$$

$$f_{3} = \frac{1}{2}(\sigma_{1}' - \sigma_{2}') + \frac{1}{2}(\sigma_{1}' + \sigma_{2}')\sin\varphi - c\cos\le 0.$$
(41)

Two parameters of the plastic model appear in the stretching functions - the friction angle  $\varphi$  and the adhesion *c*. These functions together represent a hexagonal cone of principal stresses in space, as

shown in Fig. 2.7.

When the Mohr-Coulomb model is applied to general stress states, a special treatment is required for the intersection of the two stress surfaces. Some programs use a smooth transition from one stress surface to another, that is, by rounding the corners. In "PLAXIS", however, the exact form of the full Mohr Coulomb model is implemented, using a sharp transition from one stress surface to another.

When c>0, the stress state is determined by the standard Mohr-Coulomb criterion. In fact, the allowable tensile stresses increase with coupling. This behavior can be analyzed by "PLAXIS" by determining another solution of the stress state. In this case, Mohr circles with negative head pressure cannot be. Another stress state method introduces an additional three reduced functions, which are defined as

$$f_{4} = \sigma'_{1} - \sigma_{1} \le 0$$

$$f_{5} = \sigma'_{2} - \sigma_{1} \le 0$$

$$f_{5} = \sigma'_{3} - \sigma_{1} \le 0$$
(42)

When this stress state procedure is used, the allowable stress, the bound strain function,  $\sigma$  is assumed to be zero. The flow rules are followed for the three stresses. For the stress state within the reduced surface, the elastic behavior obeys Hooke's law for an isotropic linear elastic medium.

The basic principle of elastic plasticity is that deformations and stresses have elastic and plastic parts  $\overline{\varepsilon} = \overline{\varepsilon}^e + \overline{\varepsilon}^p$ ,

$$\overline{\dot{\varepsilon}} = \overline{\dot{\varepsilon}}^e + \overline{\dot{\varepsilon}}^p, \qquad (43)$$

Hooke's law for the relationship between stresses and elastic strains

$$\overline{\dot{\sigma}}^{,} = \overline{\overline{D}}^{\,e} \overline{\dot{\varepsilon}}^{\,e} = \overline{\overline{D}}^{\,e} (\,\overline{\dot{\varepsilon}} - \overline{\dot{\varepsilon}}^{\,p}\,), \tag{44}$$

In the general case, plastic deformations will be written as:

$$\bar{\dot{\varepsilon}}^{p} = \lambda \frac{\partial g}{\partial \overline{\sigma}}, \qquad (45)$$

 $\overline{\dot{\varepsilon}}^{p}$  - plasticity parameter. For exclusively elastic behavior, the parameter is zero, while in the case of plastic behavior it is positive

$$\lambda = 0 \text{ for } f < 0 \text{ or } \frac{\partial f^{T}}{\partial \overline{\sigma}}, \overline{D}^{e} \overline{\dot{\varepsilon}} \le 0, \text{ elasticity}$$
$$\lambda > 0 \text{ for } f = 0 \text{ and } \frac{\partial f^{T}}{\partial \overline{\sigma}}, \overline{D}^{e} \overline{\dot{\varepsilon}} > 0, \text{ plasticity}$$

These equations may be used to derive the following relationship between effective stresses and stresses for an elastoplastic problem

$$\overline{\dot{\sigma}}^{\,,} = (\overline{\overline{D}}^{\,e} - \frac{\alpha}{d} \overline{\overline{D}}^{\,e} \frac{\partial g}{\partial \overline{\sigma}^{\,,}} \frac{\partial f^{\,T}}{\partial \overline{\sigma}^{\,,}} \overline{\overline{D}}^{\,e}) \overline{\dot{\dot{\varepsilon}}}^{\,,} \tag{46}$$

$$d = \frac{\partial f^{T}}{\partial \overline{\sigma}} \overline{D}^{e} \frac{\partial g}{\partial \overline{\sigma}}, \qquad (47)$$

If the behavior of the material is elastic, then the value of  $\alpha$  is zero. If the material behaves plastically, then this value is equal to 1. The theory of plasticity mentioned above is limited to the stretching surface and does not cover the entire surface of the reduced contour presented in the Mohr-Coulomb model. For such a surface, the theory of plasticity was extended others, with two or more plastic potential functions

$$\bar{\dot{\varepsilon}}^{\,p} = \lambda_1 \frac{\partial g_1}{\partial \bar{\sigma}^{\,,}} + \lambda_2 \frac{\partial g_2}{\partial \bar{\sigma}^{\,,}} + \dots \tag{48}$$

The Mohr-Coulomb nonlinear model makes it possible to more accurately describe the behavior of the soil under loads, but due to the complexity of the mathematical apparatus, it is used only in the calculation of buildings and structures using a computer. For the analytical description of dynamic phenomena, the elastic half-space model is sufficient and allows solving engineering problems with satisfactory accuracy.

When calculating according to the software complex, it is possible to use other soil base models. The loose soil model is used to accurately analyze the logarithmic work in compression of normally consolidated loose soil. The soft creeping soil model is an improved version of the soft soil model that includes simulation of the second stage of creep. The solid model is used for harder soils such as overconsolidated clays and sands. An elastic-plastic type of hyperbolic model is used here [8,9,11]. The reinforced soil model is a model for modeling the behavior of different types of soil (both soft and hard). When under the action of a variable load on the soil, the stiffness decreases, and at the same time, irreversible plastic deformations develop. This model uses the theory of plasticity, takes into account dilatancy, and stretching of the upper part of the hyperbola.

The main characteristics of the model [10]:

- load depending on soil stiffness *m*;

- shear stiffness in the standard drained triaxial test  $E_{50}^{ref}$ ;

- tangent stiffness for initial compression load  $E_{oed}^{ref}$ ;

- elastic unloading  $E_{ur}^{ref}$ ;

- parameters: c - specific adhesion,  $\varphi$  - angle of internal friction,  $\psi$  - angle of dilatation.

The main feature of this model is the dependence of stresses on stiffness.

Assume that the triaxial load application conditions,  $\sigma_2 = \sigma_{3'}$  and  $\sigma_{1'}$  are the main compressive stresses.

The model explains both plastic and elastic deformations. Plastic deformations develop only when the initial load is applied, and elastic deformations develop both during the initial load and during unloading - reloading. For dried triaxial stress tests  $\sigma_2 = \sigma_3 = \text{constant } [5]$ .

# 2.6 Principles of calculations

Determination of the dynamic characteristics of the foundation of the machine, structure, soil is based on the definition of a dynamic equation that can be determined according to the D'Alembert principle (kinetostatics). At the same time, the sum of the acting forces and the forces of inertia is zero, or is based on Newton's second law - the law of dynamics (Fig. 6).

The obtained dynamics equation can be solved by mathematical methods. Mathematical expressions for finding dynamic characteristics of machine foundations and soil are presented in building regulations or in scientific literature. The dynamics equation can be solved by numerical methods, namely the finite element method implemented in the "PLAXIS" software complex.

When calculating the foundation of the machine using mathematical expressions of dynamic characteristics, the following must be taken into account:

- the eccentricity in the distribution of foundation masses is not taken into account, if it does not exceed 3-5% according to clause 1.8 [7];

- when several disturbing forces are acting on the foundation of the machine at the same time and there is no data on their phase relationship, variants of in-phase and anti-phase action of forces are considered, which causes the most unfavorable forms of oscillations;

- when calculating to take into account the properties of the material and soil, the deformation modulus and coefficients are used.

The process of calculating dynamic characteristics using the finite element method in "PLAXIS" is also divided into phases. In each of the phases of the calculation, a specific load is activated at a certain time of the simulation. When performing a dynamic calculation, we have the following phases:

*Phase* 1 - applying a static load to the base from the own weight of the structure or foundation.

*Phase* 2 - application of a static load acting on the structure or foundation. If there is no such load, such as when calculating the dynamic characteristics of a tailings dam, this phase is not included in the calculation.

When performing calculations in phase 1 and 2, it is possible to specify a new state that should be obtained at the end of the calculation phase. And the geometric scheme and load can also be changed.

*Phase* 3 - applying a dynamic load. We set the dynamic load and the frequency with which it is applied.

*Phase* 4 - vibration damping, when there is no dynamic load.

Each phase of the calculation is generally divided into a number of steps. The presence of steps is explained by the non-linear behavior of the soil, which requires loads that are applied in small parts (step by step).

The value of the quantity calculated in the previous steps of the calculation can be ignored at the beginning of the current phase of the calculation. In this way, the beginning of the calculation is carried out from the zero field. If this function is not selected, then

step values calculated in the current phase of the calculation process will be added to those calculated in the previous phase.

Each phase in the calculation has a specific time interval. The program can work in the mode of automatic selection of size increment or time step. The calculation end time is calculated automatically by adding time intervals to all successive phases.

The calculation step used in the dynamic calculation is constant and is equal to  $\delta t = \Delta t / (n \cdot m)$ : where  $\Delta t$  is the duration of the dynamic load (time interval), *n* is the number of additional steps and *m* is the number of dynamic substeps. For each additional step (from 1 to 250, set in the program 100), the program calculates the number of substeps needed to reach the end time and evaluates  $\delta t_{crit}$ . If the calculation scheme contains very small elements, then the number of substeps can be large, which will lead to a decrease in the accuracy of calculations. Then you need to change the number of substeps.

During calculations, it is possible to constantly adjust the grid of finite elements in the calculation process. In some cases, the usual calculation of small deformations can show a significant change in the mesh geometry. In this case, it is advisable to perform the calculation on a variable grid.

"PLAXIS" has advanced capabilities for graphical presentation of calculation results. The exact values of the calculated value are entered in the output tables. All data can be output to a printer or plotter in tabular or full-color format. Graphic output of the deformed mesh, general or discrete vibration displacements, vibration velocities, vibration acceleration is carried out. "PLAXIS" allows you to create graphs of all dynamic characteristics at any point of the calculation scheme [5].

## 3. Conclusions

1. The application of the mathematical and numerical method of finite elements in dynamic calculations of structures, buildings, and soil allows to solve the following problems with a certain accuracy: to determine the vibration displacement, vibration acceleration, vibration speed of structures, soil; to find the amount of dynamic settlement of foundations depending on certain soil conditions.

2. The use of a mathematical method in dynamic calculations allows to describe mathematically with the use of certain assumptions and to determine the dynamic characteristics of the object's oscillations and the settlement of the foundation, which is investigated using formulas. And the software complex using the numerical method of finite elements makes it possible to obtain a complete picture of the field of vibration displacements, vibration accelerations, vibration velocities by obtaining graphs of the dependence of dynamic characteristics at any point of the calculation scheme on time, graphical output of the deformed grid.

3. When calculating dynamic characteristics using the analytical method, an elastic model of the soil environment is used. And when calculating according to the finite element method, a static calculation was first performed, where the basis is considered physically and geometrically nonlinear, taking into account dilatancy, etc. Then a dynamic calculation was carried out, where the behavior of the base was assumed to be elastic.

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