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STRESSED – DEFORMED STATE OF DAM FOUNDATIONS MADE OF SOIL MATERIAL WITH A TRAPEZOIDAL TRANSVERSE PROFILE

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Abstract

The mining complex is a dangerous source of environmental destruction and environmental pollution. This is especially evident in the case of large-scale mining operations.

Typical technogenic landscapes were formed in the course of open and underground mining of coal, uranium, alumina, iron ore and other mineral deposits in the territories of Donetsk, Lugansk and other regions of Ukraine, where mining and ore-dressing enterprises are located. The markers of such landscapes are the presence of heaps, dumps, industrial waste dumps, sludge pits and other sources of environmental pollution.

It is no coincidence that extractive industries are often areas of ecological disaster. The specifics of the impact of the mining complex depends on the method of extraction (open or closed), the resource being mined, and the natural features of the territory where the extraction takes place. The following main directions of the impact of extractive industries on nature and humans are distinguished:

- damage to land, the formation of anthropogenic landforms;
- change in the water balance of the territory;
- dusting of the atmosphere associated with blasting during open pit mining;
- change in the entire landscape, the formation of so-called technogenic landscapes, characterized by the almost complete absence of soil cover, vegetation, and microorganisms.

Under the conditions of Ukraine, the greatest harm to the environment is caused by the enterprises of the coal industry. The main factors of the coal industry that affect the state of the environment include:

- withdrawal from circulation of lands, their pollution with waste from coal mining and enrichment;
- depletion of water resources and changes in the hydrological regime of ground and surface waters;
- pollution of underground and surface water bodies by industrial and domestic wastewater from enterprises and settlements;
- pollution of the air basin with solid and gaseous harmful substances during the extraction, processing and combustion of solid fuels (numerous boiler houses, smoking rock dumps, etc.).

One of the ways to solve the water problem of extractive regions is the construction of artificial reservoirs, the main element of which is dams. At the same time, dams made of soil materials are most widely used. The research materials presented in this paper are aimed at solving the problem of calculating and designing reservoir dams from soil materials.

Introduction

Currently, soil dams are used for recreation in mining regions, for communications, electricity production, etc. In addition, they are used for the needs of agriculture (for example, for the construction of reservoirs).

However, when designing soil dams on foundations made of weak soils, such problems sometimes arise:

1. Sometimes the subsidence of dams is of the same order as their height [1,2,3]. In this, it is necessary to know the subsidence of the base of the dam not only in its center, but also the subsidence of characteristic points along its entire sole. This is important in order to achieve compliance of the dam profile with its design state, taking into account base deformations.

2. Foundations of soil dams made of weak soils have low strength. Therefore, it is necessary to be able to assess the strength of soil foundations of soil dams. This assessment can be carried out only if the properties of the soil and acting stresses in the base are known [1,2,3,4].

3. The profile of soil dams usually has a trapezoidal shape. In addition, usually the dam includes elements with different specific gravity (usually it is the soil of the dam body and its core) [3,4,5,6,7]. Therefore, the load on the base differs from that shown in fig. 1 and fig. 2.

Also, in [1,3,4], the coefficient of attenuation of additional stresses along the depth (using it to calculate the vertical normal stresses along the depth of the foundation) is given in tabular form only for a load having a rectangular shape (in analytical form in regulatory documents, these data are also not presented).

There are very general and approximate instructions in this regard in the construction regulations currently in force in Ukraine and other countries regarding the accounting of the specified factors [1,2,3,4,5, 6,7,8].

A similar picture occurs in the scientific and technical literature devoted to this problem [9,10]. The theses set forth in this paragraph (i.e. in paragraph 3) allowed us to conclude that the use of construction standards in force on the territory of Ukraine leads to an insufficiently accurate calculation of the stresses acting on the foundation due to the weight of the dam.

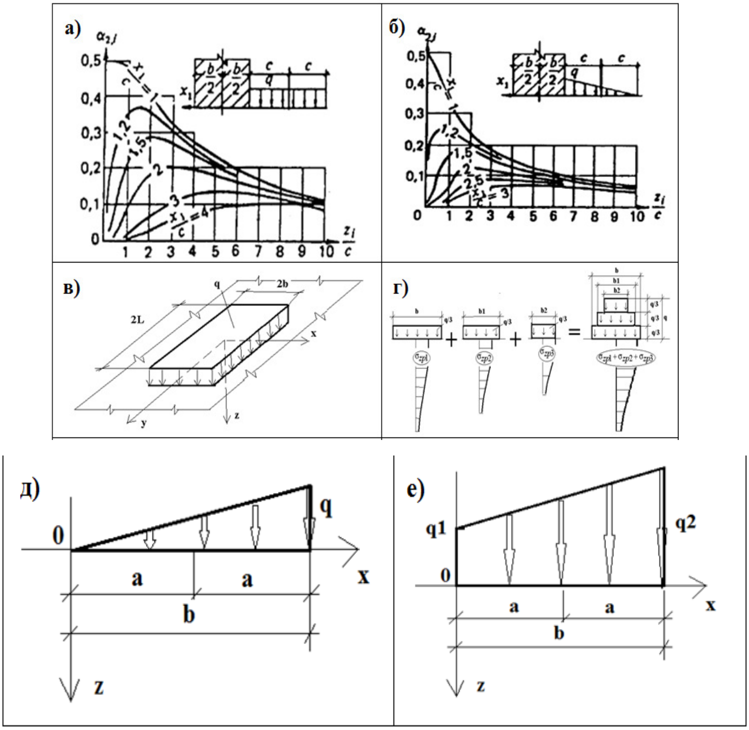

Fig. 1. The types of loads used to determine stresses in soil foundations are listed in regulatory documents and technical literature: *а* and *б* - loads on the base considered in [3,4]; *в* - the same, in [1,2]; *д* - and *е* - the same, in [10]

4. Recently, the finite element method has become widely used to predict the stress-strain state of soil foundations [11,12]. However, in this case, the calculation results are significantly affected by the dimensions of the calculation area of the base. Since there are no instructions on this matter in the regulatory documents of Ukraine, the data obtained during the forecast of the stress-strain state of the foundations of soil dams require verification and, if necessary, correction.

Therefore, when writing this monograph, we aimed to find a solution to these problems by constructing accurate analytical dependences of stresses on the coordinates and the trapezoidal external load applied to the upper limit of the base.

Application of trapezoidal vertical load

Currently, the following types of loading are used to determine the stress-deformed state of foundations of soil dams (fig. 1), the use of which is not always possible to solve the problem we mentioned. Therefore, the research task was formulated as follows:

1. An unfavorable case of a stressed - deformed state of the foundation - a dam made of soil material, the length of which is significantly greater than its width and height (fig. 2) is considered.

2. The transverse profile of the dam has the form of a trapezoid. At the same time, there may be materials with different specific gravity within the dam (for example, the core and body of the dam).

3. The pressure from the weight of the dam on the base is equal to the product of the specific gravity of its material by a height equal distance from the sole of the dam to the day surface of the dam at the vertical.

Necessary:

1. Within the framework of the model of a linear elastic isotropic medium, the conditions of plane strain and the one presented in fig. 2 loads to obtain accurate analytical solutions necessary

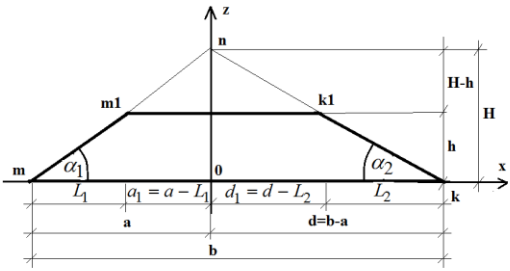

Fig. 2. Determination the geometric dimensions of the soil dam and the load on the base from its weight for determining the stressed-strained state of soil dams

Notes:

1. Bold lines indicate the outline of dam, and thin lines indicate auxiliary structures.

2. In fig. the following designations are accepted: - *h*- height of the dam, - *b* - width of the base of the dam; b_1 - dam crest width; α_1 and α_2 - the laying angles of the left and right slopes of the dam, respectively.

3. To develop an algorithm that allows taking into account the heterogeneity of the materials from which they are made when determining the subsidence of the foundations of soil dams.

4. Develop an algorithm that allows you to take into account the heterogeneity of the structure of the soil layer when determining the subsidence of soil dams.

5. Develop an algorithm that allows you to simultaneously take into account the factors outlined in paragraphs 1 and 2.

At the first stage, the research task was formulated as follows. A trapezoidal vertical load is applied to the foundation. At an arbitrary point of the base with known coordinates, it is necessary to determine the stresses caused by this load.

To solve the problem, we used the one presented in fig. 2 scheme of the profile of the dam, using which the triangular diagram of contact pressures can be transformed into a trapezoidal load.

To do this, it is necessary to present the parameters of the dam contour in an analytical form. We have:

1. The coordinate of any point of the triangle mnk can be calculated using the formula

$$Z_{mnk}(x) = H \cdot \left\{ \left(1 + \frac{x}{a}\right)\left[U(x+a) - U(x)\right] + \left(1 + \frac{x}{d}\right)\left[U(x) - U(x-d)\right] \right\} \tag{1}$$

Here H,a and d see diagram in fig. 2, and $U(x)$ - Heaviside step function [13].

2. The coordinate of any point of the m_1nk_1 triangle on the height interval $z \in (0, H\text{-}h)$ can be calculated using the formula

$$Z_{m_1nk_1}(x) = (H-h) \cdot \left\{ \left(1 + \frac{x}{a_1}\right)\begin{bmatrix} U(x+a_1) - \\ -U(x) \end{bmatrix} + \left(1 + \frac{x}{d_1}\right)\begin{bmatrix} U(x) - \\ -U(x-d_1) \end{bmatrix} \right\} \tag{2}$$

Here H,h,a_1 and d_1 - see the diagram in fig. 2.

3. Using the scheme in fig. 2 and formulas (1) and (2) we find the coordinate of any point of the trapezoid mm_1k_1k.

We have

$$\begin{gathered} Z_{mm_1k_1k}(x) = Z_{mnk}(x) - Z_{m_1nk_1}(x) = \\ = H \cdot \left\{ \left(1 + \frac{x}{a}\right)\left[U(x+a) - U(x)\right] + \left(1 + \frac{x}{d}\right)\left[U(x) - U(x-d)\right] \right\} - \\ -(H-h) \cdot \left\{ \left(1 + \frac{x}{a_1}\right)\begin{bmatrix} U(x+a_1) - \\ -U(x) \end{bmatrix} + \left(1 + \frac{x}{d_1}\right)\begin{bmatrix} U(x) - \\ -U(x-d_1) \end{bmatrix} \right\} \end{gathered} \tag{3}$$

4. In order to determine the load on the base from one meter of the length of the soil dam, (3) should be multiplied by the specific weight of the dam material $\gamma_{\text{пл}}$ and 1 meter. We have

$$q(x) = \gamma_{пл} \cdot \left[Z_{mnk}(x) - Z_{m_1 n k_1}(x) \right] =$$

$$= \gamma_{пл} \cdot H \cdot \left\{ \left(1 + \frac{x}{a} \right) \left[U(x+a) - U(x) \right] + \left(1 + \frac{x}{d} \right) \left[U(x) - U(x-d) \right] \right\} -$$

$$-\gamma_{пл} \cdot (H - h) \cdot \left\{ \left(1 + \frac{x}{a_1} \right) \begin{bmatrix} U(x+a_1) - \\ -U(x) \end{bmatrix} + \left(1 + \frac{x}{d_1} \right) \begin{bmatrix} U(x) - \\ -U(x - d_1) \end{bmatrix} \right\} \quad (4)$$

To determine the dependence of the stresses acting at the base of the dam on the coordinates, we will use the well-known fundamental solution of Flaman about the vertical concentrated force applied to the horizontal upper limit of the infinite half-plane and the principle of superposition (fig. 2).

According to [14] we have

$$\left. \begin{array}{c} \sigma_z(x,z) = \dfrac{2 \cdot z^3}{\pi} \cdot \displaystyle\int_{x_1}^{x_2} \dfrac{q(\xi) \cdot d\xi}{r(x,z,\xi)^4}; \quad \sigma_x(x,z) = \dfrac{2 \cdot z}{\pi} \cdot \displaystyle\int_{x_1}^{x_2} \dfrac{q(\xi) \cdot (x-\xi)^2 \cdot d\xi}{r(x,z,\xi)^4}; \\ \tau_{xz}(x,z) = \dfrac{2 \cdot z^2}{\pi} \cdot \displaystyle\int_{x_1}^{x_2} \dfrac{q(\xi) \cdot (x-\xi) \cdot d\xi}{r(x,z,\xi)^4}; \quad r = \sqrt{(x-\xi)^2 + z^2}. \end{array} \right\} \quad (5)$$

Here $\sigma_x(x,z)$ and $\sigma_z(x,z)$- normal stresses acting in the direction of the $0x$ and $0z$ axes respectively; $\tau_{xz}(x,z)$ - tangential stresses in the plane $0xz$; $q(x)$ - load on the day surface of the base.

First, we will find the distribution of vertical normal stresses $\sigma_{z,\mathrm{mnk}}(x,z)$ at the point of the base with coordinates (x,y) from the one presented in fig. 2 load, distributed over a triangle mnk.

To do this, in formula (1), we replace the variable x with the variable ξ, substitute the expression obtained in this way into the upper formula (5) and perform the integration procedure on the interval from x_1=-a to x_2=d.

We have

$$\left. \begin{array}{c} \sigma_{z,mnk}(x,z) = \gamma \cdot H \cdot \left[\begin{array}{c} \dfrac{-x+d}{\pi \cdot d} \cdot arctg\left(\dfrac{-x+d}{z} \right) + \\ + \dfrac{x+a}{\pi \cdot a} \cdot arctg\left(\dfrac{x+a}{z} \right) - \dfrac{d+a}{\pi \cdot a \cdot d} \cdot x \cdot arctg\left(\dfrac{x}{z} \right) \end{array} \right]; \\ d = b - a. \end{array} \right\} \quad (6)$$

Next, we will find the distribution of vertical normal stresses $\sigma_{z,m1nk1}(x,z)$ at the point of the base with coordinates (x,y) from the one presented in fig. 1 load distributed over a triangle m_1nk_1.

To do this, in formula (2), we replace the variable x with the variable ξ, substitute the expression thus obtained in the upper left formula (5) and perform the integration procedure on the interval from x_1=-a to x_2=d_1.

We have

$$\left.\begin{array}{c} \sigma_{z,m_1nk_1}\left(x,z\right)=\gamma\cdot\left(H-h\right)\left[\begin{array}{l} \dfrac{-x+d_1}{\pi\cdot d_1}\cdot arctg\left(\dfrac{-x+d_1}{z}\right)+ \\ +\dfrac{x+a_1}{\pi\cdot a_1}\cdot arctg\left(\dfrac{x+a_1}{z}\right)- \\ -\dfrac{d_1+a_1}{\pi\cdot a_1\cdot d_1}\cdot x\cdot arctg\left(\dfrac{x}{z}\right) \end{array}\right]; \\ d=b-a. \end{array}\right\} \quad (7)$$

The dependence of the vertical normal stress $\sigma_z(x,z)$ on the coordinates can be found as the stress difference $\sigma_{z,mnk}(x,z)$ and $\sigma_{z,m1nk1}(x,z)$. We have

$$\left.\begin{array}{l} \sigma_z\left(x,z\right)=\sigma_{z,mnk}\left(x,z\right)-\sigma_{z,m_1nk_1}\left(x,z\right)= \\ =\gamma\cdot H\cdot\left[\begin{array}{c} \dfrac{-x+d}{\pi\cdot d}\cdot arctg\left(\dfrac{-x+d}{z}\right)+ \\ +\dfrac{x+a}{\pi\cdot a}\cdot arctg\left(\dfrac{x+a}{z}\right)-\dfrac{d+a}{\pi\cdot a\cdot d}\cdot x\cdot arctg\left(\dfrac{x}{z}\right) \end{array}\right]- \\ -\gamma\cdot\left(H-h\right)\left[\begin{array}{c} \dfrac{-x+d_1}{\pi\cdot d_1}\cdot arctg\left(\dfrac{-x+d_1}{z}\right)+ \\ +\dfrac{x+a_1}{\pi\cdot a_1}\cdot arctg\left(\dfrac{x+a_1}{z}\right)-\dfrac{d_1+a_1}{\pi\cdot a_1\cdot d_1}\cdot x\cdot arctg\left(\dfrac{x}{z}\right) \end{array}\right]; \\ d=b-a. \end{array}\right\} \quad (8)$$

If the cross section of the soil dam has the shape of an isosceles trapezoid, then d=a, d_1=b_1, b=2·a and b_1=2·a_1 , where

$$\left.\begin{aligned}\sigma_z(x,z)=\frac{\gamma\cdot H}{\pi\cdot a}\cdot\left[(x+a)\cdot arctg\left(\frac{x+a}{z}\right)-2\cdot x\cdot arctg\left(\frac{x}{z}\right)+\right.\\ \left.+(a-x)\cdot arctg\left(\frac{a-x}{z}\right)\right]-\frac{\gamma\cdot(H-h)}{\pi\cdot a_1}\cdot\left[(x+a_1)\cdot arctg\left(\frac{x+a_1}{z}\right)-\right.\\ \left.-2\cdot x\cdot arctg\left(\frac{x}{z}\right)+(a-x)\cdot arctg\left(\frac{a-x}{z}\right)\right];\quad b=2\cdot a.\end{aligned}\right\}\qquad(9)$$

Similarly, we will find normal horizontal stresses $\sigma_x(x,z)$. We have

$$\left.\begin{aligned}&\sigma_x(x,z)=\\ &=\gamma\cdot H\cdot\begin{bmatrix}-\frac{z}{\pi\cdot d}\cdot\ln\left(z^2+(-x+d)^2\right)-\frac{z}{\pi\cdot a}\cdot\ln\left(z^2+(x+a)^2\right)+\\ +\frac{z\cdot(d+a)}{\pi\cdot d\cdot a}\cdot\ln\left(x^2+z^2\right)+\frac{-x+d}{\pi\cdot d}\cdot x\cdot arctg\left(\frac{-x+d}{z}\right)+\\ +\frac{a+x}{\pi\cdot a}\cdot arctg\left(\frac{x+a}{z}\right)-\frac{d+a}{\pi\cdot d\cdot a}\cdot x\cdot arctg\left(\frac{x}{z}\right)\end{bmatrix}-\\ &-\gamma\cdot(H-h)\begin{bmatrix}-\frac{z}{\pi\cdot d_1}\cdot\ln\left(z^2+(-x+d_1)^2\right)-\frac{z}{\pi\cdot a_1}\cdot\ln\left(z^2+(x+a_1)^2\right)+\\ +\frac{z\cdot(d_1+a_1)}{\pi\cdot d_1\cdot a_1}\cdot\ln\left(x^2+z^2\right)+\frac{-x+d_1}{\pi\cdot d_1}\cdot x\cdot arctg\left(\frac{-x+d_1}{z}\right)+\\ +\frac{a_1+x}{\pi\cdot a_1}\cdot arctg\left(\frac{x+a_1}{z}\right)-\frac{d_1+a_1}{\pi\cdot d_1\cdot a_1}\cdot x\cdot arctg\left(\frac{x}{z}\right)\end{bmatrix};\\ &d=b-a;\quad d_1=b_1-a_1.\end{aligned}\right\}\qquad(10)$$

If the cross section of the soil dam has the shape of an isosceles trapezoid, then $d=a$, $d_1=b_1$, $b=2\cdot a$ and $b_1=2\cdot a_1$. Therefore, in this case, the stress $\sigma_x(x,z)$ is equal

$$\left.\begin{aligned}&\sigma_x(x,z)=\\ &=\gamma\cdot H\cdot\begin{bmatrix}-\frac{z}{\pi\cdot a}\cdot\ln\left(z^2+(-x+a)^2\right)-\frac{z}{\pi\cdot a}\cdot\ln\left(z^2+(x+a)^2\right)+\\ +\frac{2\cdot z}{\pi\cdot a}\cdot\ln\left(x^2+z^2\right)+\frac{a+x}{\pi\cdot a}\cdot arctg\left(\frac{x+a}{z}\right)-\\ -\frac{2\cdot x}{\pi\cdot a}\cdot arctg\left(\frac{x}{z}\right)+\frac{a-x}{\pi\cdot a}\cdot x\cdot arctg\left(\frac{a-x}{z}\right)\end{bmatrix}-\end{aligned}\right\}\qquad(11)$$

$$
\left.\begin{array}{c}
-\gamma\cdot\left(H-h\right)\left[\begin{array}{c}
-\dfrac{z}{\pi\cdot a_1}\cdot\ln\left(z^2+\left(-x+a_1\right)^2\right)-\dfrac{z}{\pi\cdot a_1}\cdot\ln\left(z^2+\left(x+a_1\right)^2\right)+\\
+\dfrac{2\cdot z}{\pi\cdot a_1}\cdot\ln\left(x^2+z^2\right)+\dfrac{a_1+x}{\pi\cdot a_1}\cdot arctg\left(\dfrac{x+a_1}{z}\right)-\\
-\dfrac{2\cdot x}{\pi\cdot a_1}\cdot arctg\left(\dfrac{x}{z}\right)+\dfrac{a_1-x}{\pi\cdot a_1}\cdot x\cdot arctg\left(\dfrac{a_1-x}{z}\right)
\end{array}\right];\\
b=2\cdot a;\;\; b_1=2\cdot a_1.
\end{array}\right\}\qquad(11)
$$

In a similar way, we will find the tangential stresses acting in the base $\tau_{xz}(x,z)$.We have

$$
\left.\begin{array}{c}
\tau_{xz}\left(x,z\right)=\gamma\cdot H\cdot\left[\dfrac{\left(d+a\right)\cdot z}{d\cdot\pi\cdot a}\cdot arctg\left(\dfrac{x}{z}\right)-\dfrac{z}{\pi\cdot a}\cdot arctg\left(\dfrac{x+a}{z}\right)+\right.\\
\left.+\dfrac{z}{\pi\cdot d}\cdot arctg\left(\dfrac{-x+d}{z}\right)\right]-\gamma\cdot\left(H-h\right)\cdot\left[\dfrac{\left(d_1+a_1\right)\cdot z}{d_1\cdot\pi\cdot a_1}\cdot arctg\left(\dfrac{x}{z}\right)-\right.\\
\left.-\dfrac{z}{\pi\cdot a_1}\cdot arctg\left(\dfrac{x+a_1}{z}\right)+\dfrac{z}{\pi\cdot d_1}\cdot arctg\left(\dfrac{-x+d_1}{z}\right)\right].\\
d=b-a;\;\; d_1=b_1-a_1.
\end{array}\right\}\qquad(12)
$$

If the cross section of the soil dam has the shape of an isosceles trapezoid, then $d=a$, $d_1=b_1$, $b=2\cdot a$ and $b_1=2\cdot a_1$. Therefore, in this case, the tangential stresses $\tau_{xz}(x,z)$ are equal

$$
\left.\begin{array}{c}
\tau_{xz}\left(x,z\right)=\dfrac{\gamma\cdot H}{\pi\cdot a}\cdot z\cdot\left[arctg\left(\dfrac{x}{z}\right)-arctg\left(\dfrac{x+a}{z}\right)+arctg\left(\dfrac{-x+a}{z}\right)\right]-\\
-\dfrac{\gamma\cdot\left(H-h\right)}{\pi\cdot a_1}\cdot z\cdot\left[arctg\left(\dfrac{x}{z}\right)-arctg\left(\dfrac{x+a_1}{z}\right)+arctg\left(\dfrac{-x+a_1}{z}\right)\right].\\
b=2\cdot a;\;\; b_1=2\cdot a_1.
\end{array}\right\}\qquad(13)
$$

The analytical expressions obtained by us for the stresses acting in the foundation of soil dams allow us to estimate the strength of the foundation at any point using the well-known Coulomb-Mohr criterion [9, 14, 15]. We have

$$
\left.\dfrac{\sigma_1\left(x,z\right)-\sigma_3\left(x,z\right)}{\sigma_1\left(x,z\right)+\sigma_3\left(x,z\right)-2\cdot P\left(x,z\right)+2\cdot c\cdot ctg\left(\varphi\right)}\le\sin\left(\varphi\right)\right\}\qquad(14)
$$

$$\left.\begin{array}{c}\text{if the excess pressure in the pore fluid of the base is zero;}\\ \dfrac{\sigma_1(x,z)-\sigma_3(x,z)}{\sigma_1(x,z)+\sigma_3(x,z)-2\cdot P(x,z)+2\cdot c\cdot ctg(\varphi)}\le\sin(\varphi)\\ \text{if the pressure in the pore fluid of the base is equal to } P(x,z).\end{array}\right\} \quad (14)$$

Here

$$\left.\begin{array}{l}\sigma_1(x,z)=\dfrac{\sigma_x(x,z)+\sigma_z(x,z)}{2}+\dfrac{1}{2}\cdot\sqrt{\left[\sigma_x(x,z)-\sigma_z(x,z)\right]^2+4\cdot\tau_{xz}^2(x,z)};\\ \sigma_3(x,z)=\dfrac{\sigma_x(x,z)+\sigma_z(x,z)}{2}-\dfrac{1}{2}\cdot\sqrt{\left[\sigma_x(x,z)-\sigma_z(x,z)\right]^2+4\cdot\tau_{xz}^2(x,z)}.\end{array}\right\} \quad (15)$$

where and σ_3 - the main stresses in the foundation; P - excess pressure in the pore fluid; c - specific coupling; φ - angle of internal friction; σ_x, σ_z and τ_{zx} - see the explanation of the formulas (5)-(13).

Determining the vertical movement of the base of the dam

According to formulas [9, 14], in the case of plane deformation, the vertical relative deformation of the base should be determined by the formula

$$\varepsilon_z(x,z)=\frac{1}{E}\cdot\left[(1-\nu)^2\cdot\sigma_z(x,z)-\nu(1+\nu)\cdot\sigma_x(x,z)\right] \quad (16)$$

where

$$U_z(x,z)=\frac{1}{E}\cdot\left[(1-\nu)^2\cdot\int\sigma_z(x,z)\cdot dz-\nu(1+\nu)\cdot\int\sigma_x(x,z)\cdot dz\right]+F(x) \quad (17)$$

Here - $\varepsilon_z(x,z)$ vertical relative deformation of the base; $U_z(x,z)$ - the same, displacement; E and ν - elastic technical constants of the base (modulus of elasticity and Poisson's ratio, respectively); $\sigma_z(x,z)$ and $\sigma_x(x,z)$ - vertical and horizontal normal stresses; $F(x)$ - some arbitrary function of the coordinate x.

When calculating subsidence, the method of layer-by-layer summation is currently used [1,2,3-9]. Therefore, we will find the subsidence of the base on the depth interval $z\in(z_1,\ z_2)$. For the convenience of presenting the material, we will put

$$I_{z,1}(x,z)=\frac{(1-\nu)^2}{E}\cdot\int\sigma_z(x,z)\cdot dz;\quad I_{z,2}(x,z)=\frac{\nu(1+\nu)}{E}\cdot\int\sigma_x(x,z)\cdot dz \quad (18)$$

Then the subsidence $S_i(x)$ of a layer of thickness $h_i=z_{2,i}-x_{1,i}$ and $z_{2,i}\geq z_{1,i}$ will be equal to

$$\left.\begin{aligned}
&S_i(x)=U_z\left(x,z_{2,i}\right)-U_z\left(x,z_{1,i}\right);\;\; U_z(x,z)=I_{z,1}(x,z)+I_{z,2}(x,z);\\
&I_{z,1}(x,z)=\frac{(1-\nu)^2}{E}\cdot\int\sigma_z(x,z)\cdot dz;\;\; I_{z,2}(x,z)=\frac{\nu(1+\nu)}{E}\cdot\int\sigma_x(x,z)\cdot dz;\\
&\int\sigma_z(x,z)\cdot dz=\frac{x^2\cdot(a+d)}{\pi\cdot a\cdot d}\cdot\ln\left(\frac{x}{z}\right)+\frac{(x+a)}{\pi\cdot a}\cdot arctg\left(\frac{x+a}{z}\right)+\\
&+\frac{(d-x)^2}{2\cdot\pi\cdot d}\cdot\ln\left[1+\frac{(-x+d)^2}{z^2}\right]-\frac{(d-x)^2}{\pi\cdot d}\cdot\ln\left[\frac{-x+d}{z}\right]-\\
&-\frac{x^2\cdot(d+a)}{2\cdot\pi\cdot d\cdot a}\cdot\ln\left(1+\frac{x^2}{z^2}\right)-\frac{x\cdot z\cdot(d+a)}{\pi\cdot d\cdot a}\cdot arctg\left(\frac{x}{z}\right)+\\
&+\frac{(x+a)^2}{2\cdot a\cdot\pi}\cdot\ln\left[1+\frac{(x+a)^2}{z^2}\right]-\frac{(x+a)^2}{a\cdot\pi}\cdot\ln\left[\frac{x+a}{z}\right]+\\
&+\frac{(-x+d)}{\pi\cdot d}\cdot arctg\left(\frac{-x+d}{z}\right);\\
&\int\sigma_x(x,z)\cdot dz=\frac{1}{2}\cdot\frac{(a+d)\cdot\left(x^2+z^2\right)\cdot\ln\left(x^2+z^2\right)}{\pi\cdot a\cdot d}-\\
&-\frac{1}{2}\cdot\frac{\left[z^2+(-x+d)^2\right]\cdot\ln\left[z^2+(-x+d)^2\right]}{\pi\cdot d}-\frac{1}{2}\cdot\frac{\left[z^2+(x+a)^2\right]\cdot\ln\left[z^2+(x+a)^2\right]}{\pi\cdot a}+\\
&+\frac{x^2\cdot(d+a)\cdot\ln(x/z)}{\pi\cdot a\cdot d}+\frac{x+a}{a\cdot\pi}\cdot\arctan\left(\frac{x+a}{z}\right)+\frac{d+a}{2\cdot\pi}-\\
&-\frac{(x+a)^2}{\pi\cdot a}\cdot\ln\left(\frac{x+a}{z}\right)-\frac{(-x+d)^2}{\pi\cdot d}\cdot\ln\left(\frac{-x+d}{z}\right)+
\end{aligned}\right\}\quad(19)$$

$$\left.\begin{aligned}
&+\frac{(x+a)^2}{2\cdot\pi\cdot a}\cdot\ln\left[1+\frac{(x+a)^2}{z^2}\right]+\frac{(-x+d)^2}{2\cdot\pi\cdot d}\cdot\ln\left[1+\frac{(-x+d)^2}{z^2}\right]-\\
&-\frac{x\cdot z\cdot(d+a)}{a\cdot\pi\cdot d}\cdot arctg\left(\frac{x}{z}\right)-\frac{x^2\cdot(d+a)}{2\cdot a\cdot\pi\cdot d}\cdot arctg\left(1+\frac{x^2}{z^2}\right)+\\
&+\frac{z\cdot(-x+d)}{\pi\cdot d}\cdot arctg\left(\frac{-x+d}{z}\right);\quad d=b-a.
\end{aligned}\right\}\quad(19)$$

If the transverse profile of the dam has the shape of an isosceles trapezoid, then in (19) should be put $d=a$.

Formula (19) makes it possible to use the well-known method of layer summation and the DBN formula to calculate the settlement of the foundations of soil dams due to additional stresses from the weight of the dam, namely

$$S(x) = \sum_{i=1}^{n} S_i(x), \tag{20}$$

where $S(x)\sum_{i-1}^{n} S_i(x)$ - settlement of the base of the dam at the point with the coordinate (see formula (19)), and $S_i(x)$ - settlement of the elementary layer of the base with thickness h_i, whose covering is at depth $z_{1,i}$, and the sole on the deep $z_{2,i}$.

If you do not need a very high accuracy of determining the settlement of the dam base, then the settlement due to additional stresses from the weight of the dam can be calculated using the well-known DBN formula

$$\left.\begin{aligned} S(x) = \beta \cdot \sum_{i=1}^{n} \frac{\sigma_z\left(x, z_{1,i}\right) + \sigma_z\left(x, z_{2,i}\right)}{2 \cdot E_i} \cdot h_i; \\ h_i = z_{2,i} - z_{1,i}. \end{aligned}\right\} \tag{21}$$

Here $S(x)$, $z_{1,i}$, $z_{2,i}$ - see explanation of the formula (20), σ_z - additional stress in the base, which should be determined by formulas (8) and (9), β=0,8, and E_i - the modulus of the general deformation of i - th soil layer at the depth interval. Finally, in the third part of this study, we present one of the options for using the theoretical results obtained by us regarding the determination of the stressed-strained state of an soil dam constructed of heterogeneous materials. An illustration of our proposed approach for this purpose is presented in fig. 3.

It follows from the figure that if it is necessary to calculate the stressed-strained state of the base of an soil dam in which the dam body has a specific weight γ_1, and the core - γ_1, then you should proceed as follows:

1. To perform calculations, you should use the calculation scheme in fig. 2. Herewith:

1.1. If you need to determine vertical stresses σ_z, you should use formulas (8) and (9).

1.2. If you need to determine horizontal stresses σ_x, you should use formulas (10) and (11).

1.3. If you need to determine tangential stresses τ_{zx}, you should use formulas (12) and (13).

1.4. If it is necessary to determine the settlement (i.e.vertical movements) of the base of the dam, formulas (17), (18), and (19) should be used.

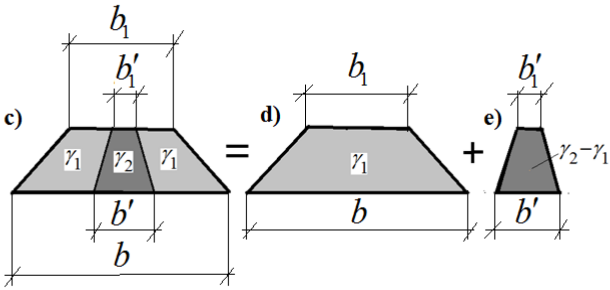

Fig. 3. The scheme of using the theoretical results obtained by us for the calculation of base of soil dams from materials with different specific gravity

Notes:

1. γ_1 - specific gravity of the dam body; γ_1 - specific gravity of the dam core.

2. The heights of the dam elements with different specific gravity are the same.

2. To determine each of the stress and displacement components listed in p.1 of this algorithm in relation to the one presented in fig. 3-c of the scheme, the following actions should be performed:

2.1. First, you should calculate the values of stresses and displacements for the one shown in fig. 3-*d* of the calculation scheme. At the same time, the specific weight of the dam material should be taken as equal γ_1.

2.2. Next, you should calculate the values of stresses and displacements for the shown in fig. 3-*e* calculation scheme. At the same time, the specific weight of the dam material should be taken as equal γ_2-γ_1.

2.3. The results obtained in this way should be added up.

Conclusions

In general, this monograph presents the following results:

1. Within the framework of the model of base in the form of a linear elastic isotropic habitat and planar deformation calculation scheme obtained analytical dependences of stresses and deformations of the half-plane, to the upper limit of which a trapezoidal load is applied in the form of an equilateral and an equilateral trapezoid.

These data are necessary for the calculation for the first and second groups of limit states of base of dams made of soil materials, embankments of railways and highways, and other structures located on foundations composed of weak soils.

2. Algorithms for determining the subsidence of the sole of soil dams due to additional stresses in the base due to the weight of the dam are proposed.

3. An algorithm for determining the stress-strain state of base of soil dams,made of materials with different specific gravity has been developed.

In conclusion, it should be noted that the research materials presented in this monograph should be used in the design of transport, energy and agricultural facilities.

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