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METHODICAL GUIDELINES

to performance of practical tasks in the discipline

«DESCRIPTIVE GEOMETRY»

for higher education students of the first (bachelor) level
in the field of study 191 «Architecture and Urban planning»
of full-time education form.

INTERSECTION OF PLANES SET BY TRACES

МЕТОДИЧНІ ВКАЗІВКИ

до виконання практичних робіт з навчальної дисципліни

«НАРИСНА ГЕОМЕТРІЯ»

для здобувачів вищої освіти першого (бакалаврського) рівня
за спеціальністю 191 «Архітектура та містобудування»
денної форми навчання.

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Методичні вказівки до виконання практичних робіт з навчальної дисципліни «Нарисна геометрія» для здобувачів вищої освіти першого (бакалаврського) рівня за освітньо-професійною програмою «Архітектура та містобудування» спеціальності 191 «Архітектура та містобудування» денної форми навчання. Перетин площин, заданих слідами [Електронне видання] / Пугачов Є. В., Літніцький С. І., Кундрат Т. М., Крівцов В. В., Зданевич В. А. – Рівне : НУВГП, 2025. – 27 с.

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INTRODUCTION

Methodical guidelines to performance of practical tasks in the discipline «Descriptive geometry» are developed according to the syllabus of the course for higher education students of the first (bachelor) level in the field of study 191 «Architecture and Urban planning» of full-time education form.

Methodical guidelines contain the main theoretical data, basic data to task; the technique of creation is described and examples of definition of straight line of intersection of two planes set by traces or triangle and traces are given, questions for independent preparation and the list of the recommended literature are given.

Tasks are performed by pencil on A3 format (album orientation). Basic data and results are allocated with the reinforced line. Auxiliary constructions show light line firm pencil.

ВСТУП

Методичні вказівки до виконання практичних робіт з навчальної дисципліни «Нарисна геометрія» розроблені відповідно до силабусу для здобувачів вищої освіти першого (бакалаврського) рівня за освітньо-професійною програмою «Архітектура та містобудування» спеціальності 191 «Архітектура та містобудування» денної форми навчання.

Методичні вказівки містять основні теоретичні відомості, вихідні дані до завдання, методику побудови та приклади побудови прямої перетину двох площини, заданих слідами або трикутником і слідами, питання для самостійної підготовки, список рекомендованої літератури.

Завдання виконуються олівцем на А3 форматі альбомної орієнтації. Вихідні дані та результати виділяються потовщеною лінією. Допоміжні побудови показують тонкою лінією твердим олівцем.

1. MAIN THEORETICAL DATA

The provision of the plane in space can be set by traces. **Traces of the plane** are called lines of intersection of the plane with the projection planes (Fig. 1).

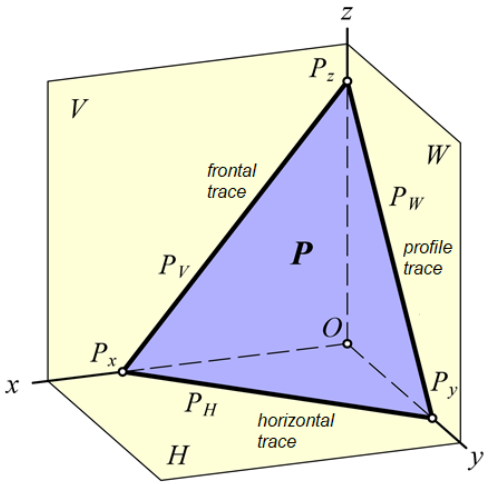


Fig. 1

Generally the plane has three traces (Fig. 1): **the horizontal** trace P_H is the intersection of the plane P with the horizontal projection plane H; **the frontal** trace P_V is the intersection of the plane P with the frontal projection plane V; **the profile** trace P_W is the intersection of the plane P with the profile projection plane W. The traces of the plane P intersect in pairs on the axes at points Px, Py, Pz which are called the origin points of the plane traces (Fig. 1, 2). In Fig. 2 traces and points of an origin of traces of the same plane P on the diagram are represented.

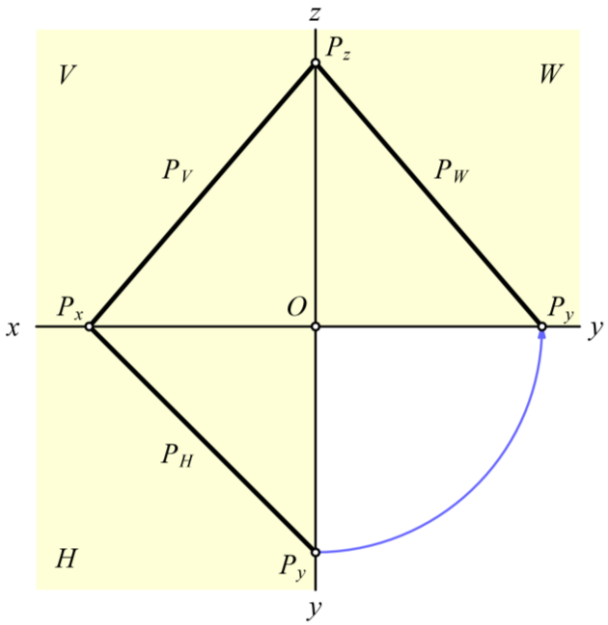


Fig. 2

Creation of the line of intersection of the planes is one of the main objectives of descriptive geometry.

At problem solving of intersection of two planes the following cases are possible:

– both planes are projecting rather same plane of projections;

– one of the planes is projecting and another is of the general provision;

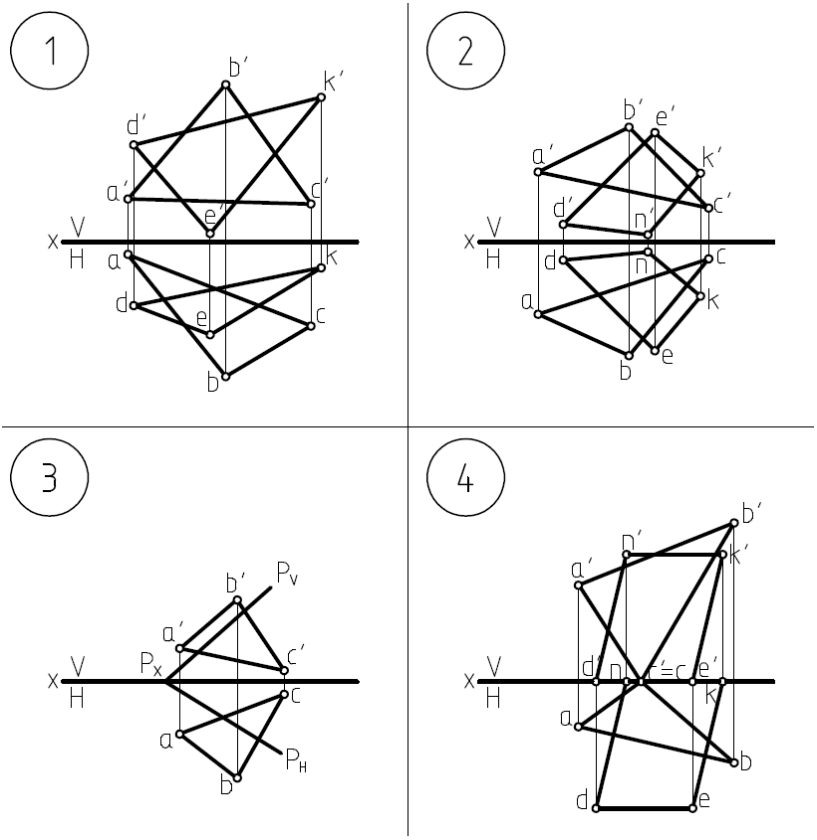
– both planes of the general provision.

2. INPUT DATA FOR CREATION OF INTERSECTION OF TWO PLANES

Task. Construct a straight line of intersection of two planes given in different ways.

Input data to the task get out of table 1 according to number of option which matches number one after another in the list of group.

Table 1



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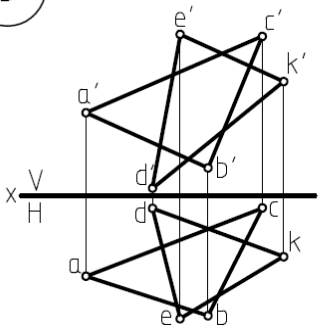
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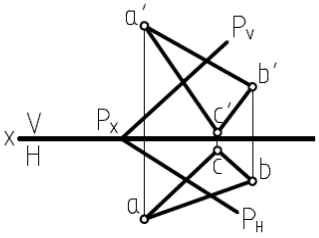
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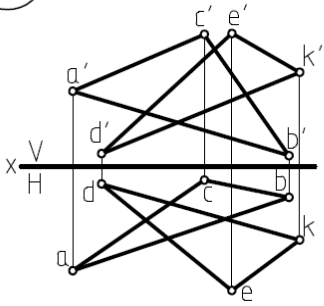
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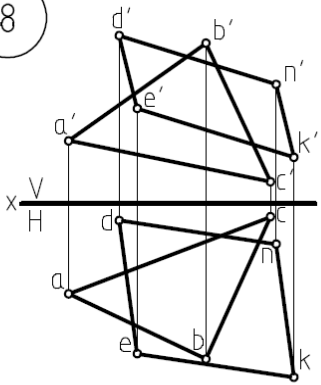
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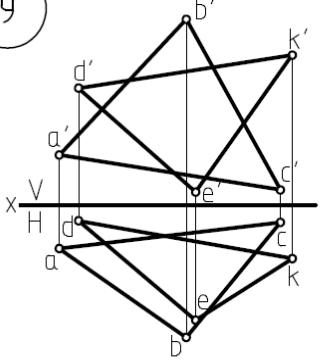
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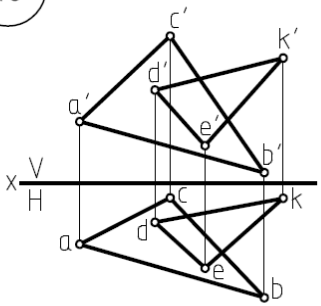
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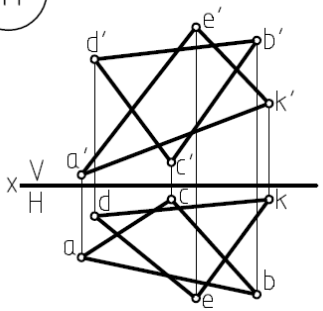

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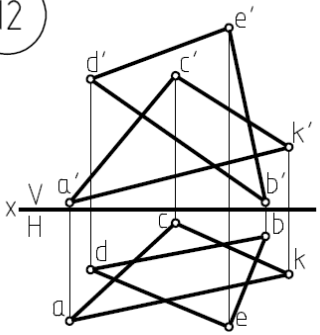

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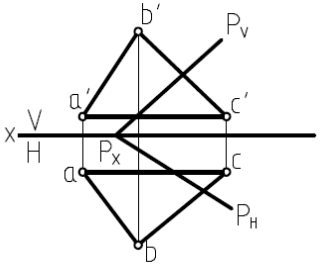

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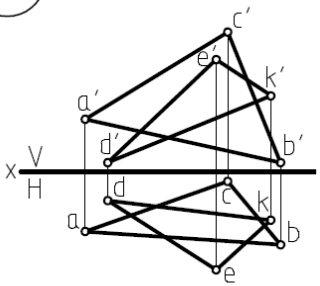

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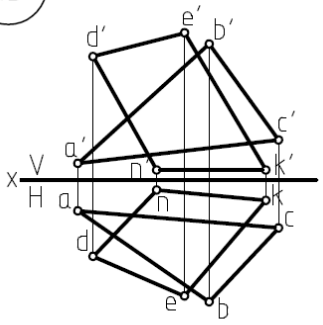

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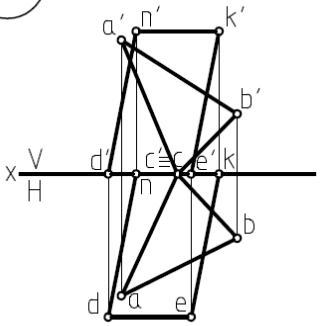

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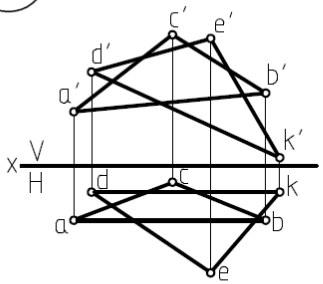

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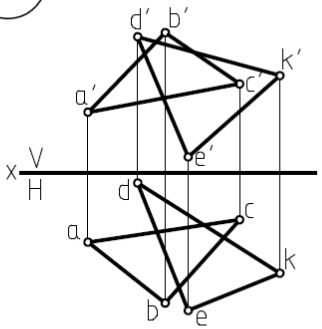

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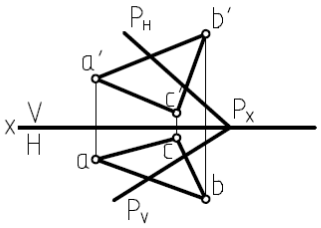

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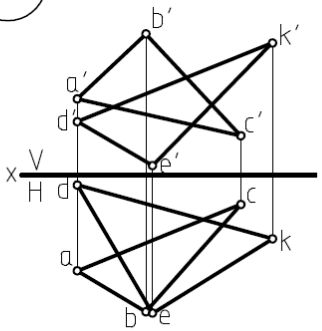

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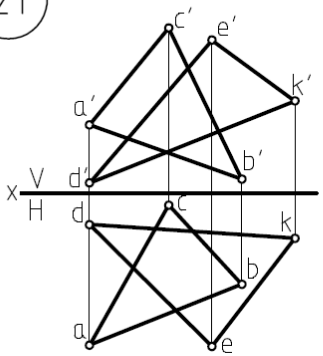

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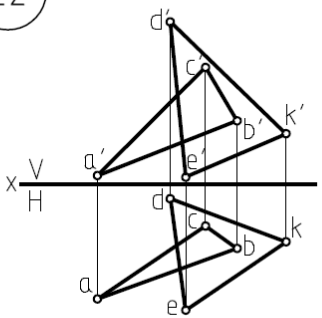

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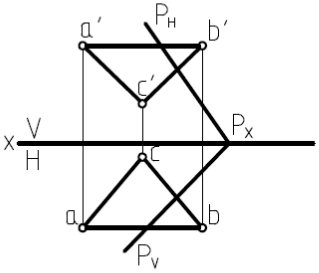

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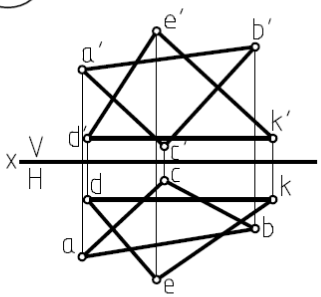

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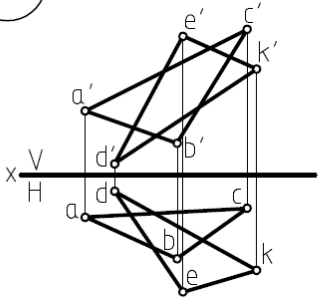

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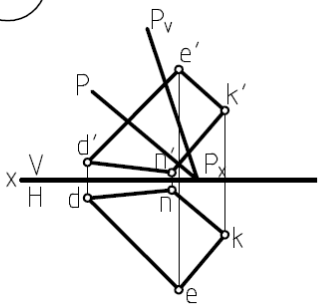

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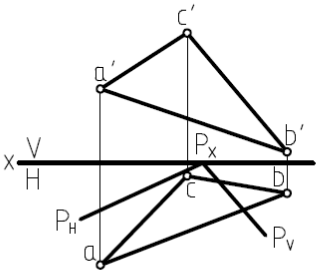

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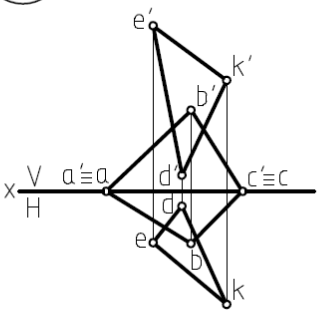


3. METHODICAL GUIDELINES

3.1. PLANES SET BY TRACES

Let planes ***P*** and ***S*** be set by their frontal and horizontal traces (fig. 3). To define a straight line of intersection of the planes.

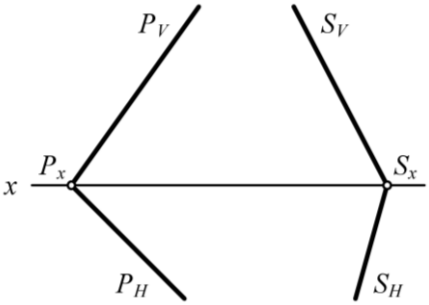


Fig. 3

Two planes ***P*** and ***S*** intersect along a straight line, shown in Fig. 4 by a thick blue line, for the construction of which it is necessary to determine two points ***1*** and ***2***.

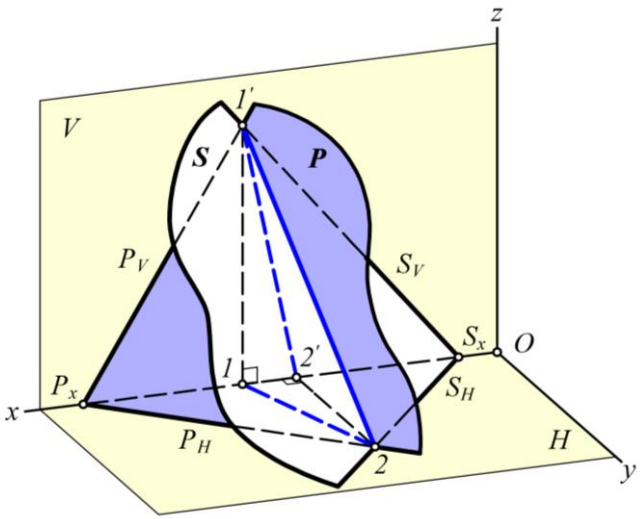


Fig. 4

In Fig. 5, *a*, the frontal traces of the planes intersect at point ***1'***, and the horizontal traces intersect at point ***2***.

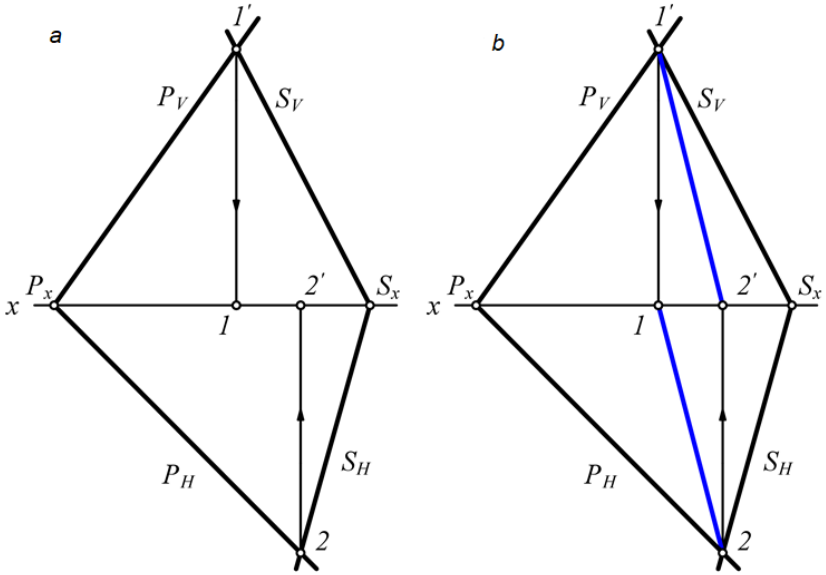


Fig. 5

Since point ***1'*** lies in the frontal plane of projections, its horizontal projection (point ***1***) lies on the axis ***x***. Respectively the point ***2*** lies in the horizontal plane of projections, and its frontal projection (a point ***2'***) lies on axis ***x*** too. Therefore, we have the line ***12*** of intersection of the planes ***P*** and ***S***, shown in Fig. 5, *b* by a thick blue line.

If traces of the same name do not intersect within the drawing, as shown in Fig. 6, then they can be intersected by cutting planes, namely **the method of auxiliary cutting planes**. It is better to use level planes, because then there will be level lines (frontal or horizontal) in the intersection.

In Fig. 6, the planes ***P*** and ***S*** are intersected by horizontal cutting planes ***R*** (Fig. 6, *a*) and ***T*** (Fig. 6, *b*).

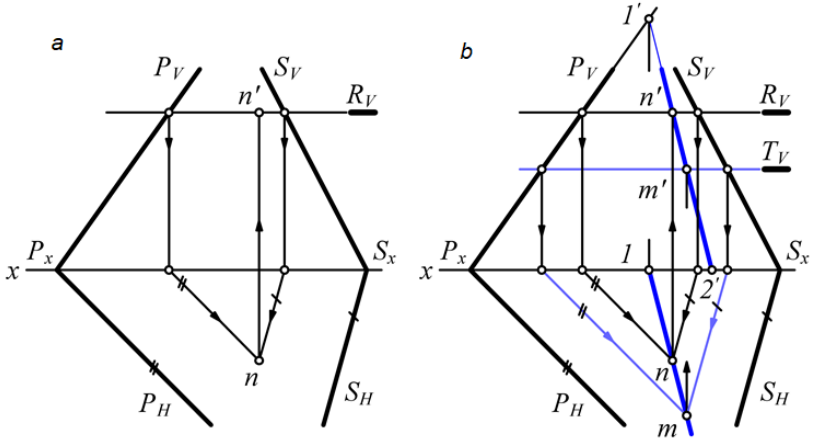


Fig. 6

The frontal traces $\boldsymbol{R_V}$ and $\boldsymbol{T_V}$ intersecting with the frontal traces $\boldsymbol{P_V}$ and $\boldsymbol{S_V}$ of the given planes give the frontal projections of the points whose horizontal projections are incident on the axis ***x***. Through them there pass horizontal projections of horizontals according to which the ***R*** and ***T*** planes intersect the given planes. The corresponding pairs of horizontal projections of the horizontals in the section determine the line of intersection ***MN***, which, of course, coincides with the line of intersection ***12***, which was determined in the manner described above.

The problem can also be solved using auxiliary level cutting planes, but not horizontal ones, but frontal ones.

Fig. 7, *a* shows how, instead of an auxiliary level plane, an auxiliary secant plane of general position $\boldsymbol{S_1}$, parallel to a given plane ***S***, can be used. It intersects the given plane ***P*** along a straight line parallel to the line of intersection of the given planes ***P*** and ***S*** (Fig. 7, *b*). Having determined the direction of the frontal and horizontal projections of the straight section, determine the positions of points ***1*** and ***2'*** (segment $\boldsymbol{P_xS_x}$) using the ratio in which points $\boldsymbol{1_1}$ and $\boldsymbol{2_1'}$ divide the segment $\boldsymbol{P_xS_{1x}}$.

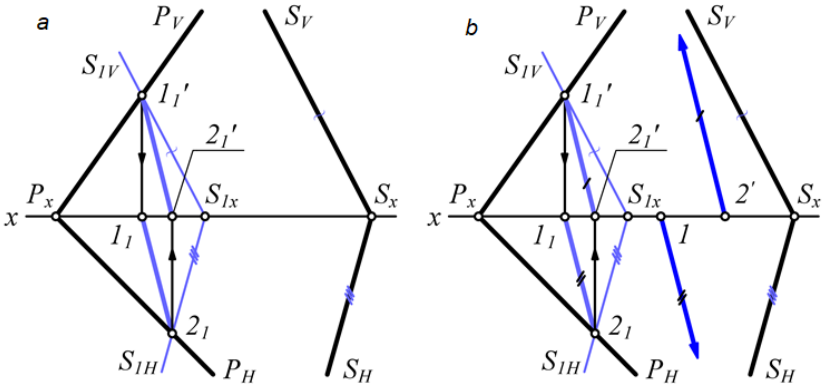


Fig. 7

If one of the planes set by traces holds the projecting position (plane ***S***), then the projection of the line of intersection of the ***P*** and ***S*** planes will coincide with the corresponding trace of the projecting plane (in Fig. 8 – with a horizontal trace of $\boldsymbol{S_H}$ of plane ***S***).

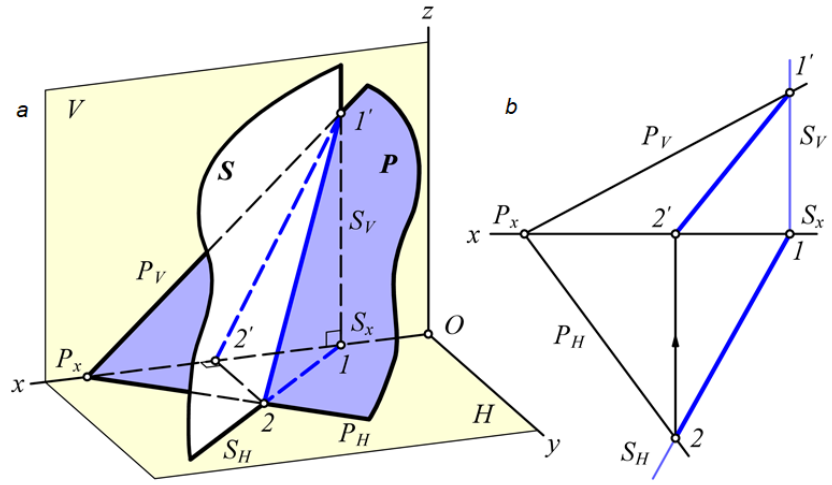


Fig. 8

Note that when the plane ***S*** is rotated relative to the straight line of intersection of the planes, the position of the origin point of the traces ***Sx***, the frontal and horizontal traces of the plane ***S*** changes, but the projection of the straight line of intersection onto the frontal and horizontal planes of projections remains unchanged (thickened blue line in Fig. 8, Fig. 9, *a*; *b*).

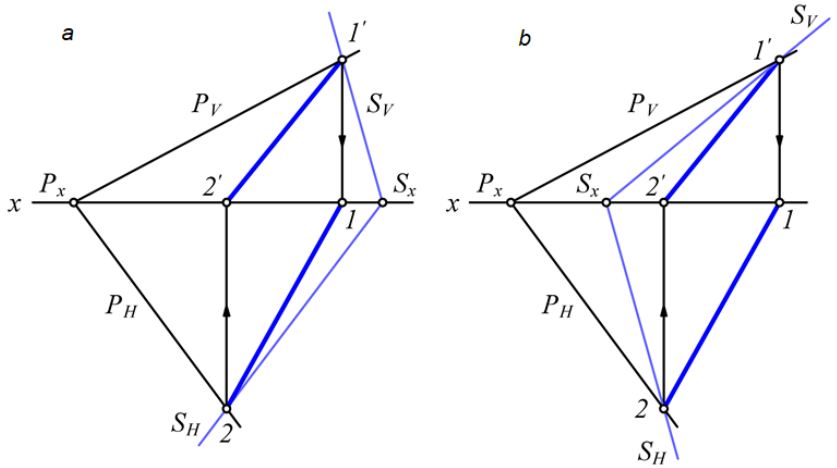


Fig. 9

Fig. 10 shows the construction of the line of intersection of two horizontally projecting planes given by traces, and in Fig. 10, *b* the horizontal traces of the planes ***P*** and ***S*** intersect above the axis x.

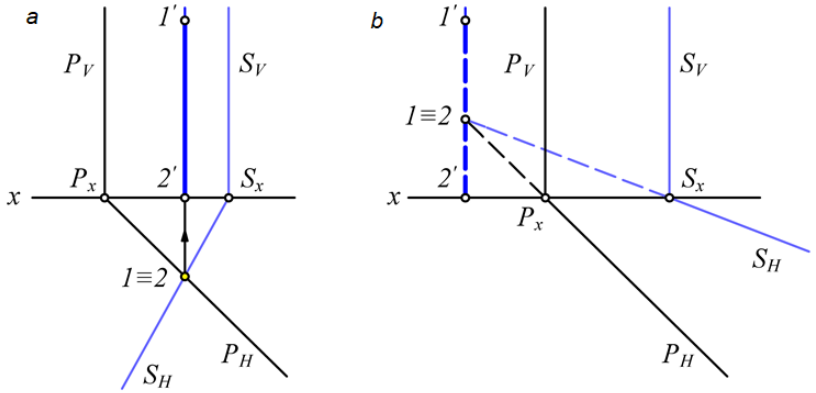


Fig. 10

Fig. 11 shows the intersection of two planes ***P*** and ***S***, whose horizontal traces are parallel. The line of intersection is a horizontal line parallel to the horizontal traces of the planes (in the figure, the blue line).

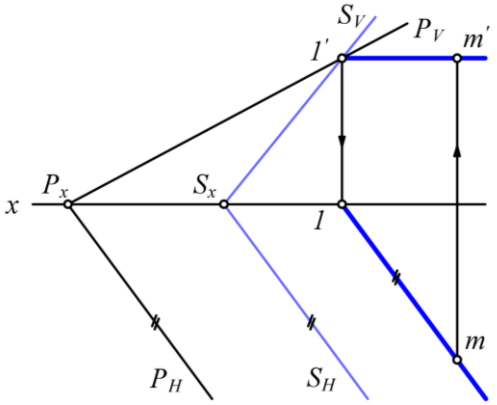


Fig. 11

Fig. 12 shows the intersection of two planes of general position given by traces whose horizontal traces intersect above the axis ***x***.

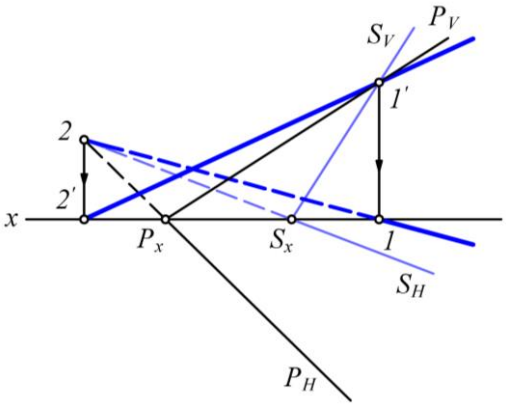


Fig. 12

In fig. 13 for definition of a straight line of intersection ***the method of auxiliary oblique-angled projection*** is used. The projection occurs in the direction of the frontal trace S_V of the plane S. Since the horizontal projection of the frontal trace S_V coincides with the axis x, the horizontal projection of the projection direction will be S_0. With this direction of projection, the plane S is projected onto its horizontal trace S_H, that is, into a straight line.

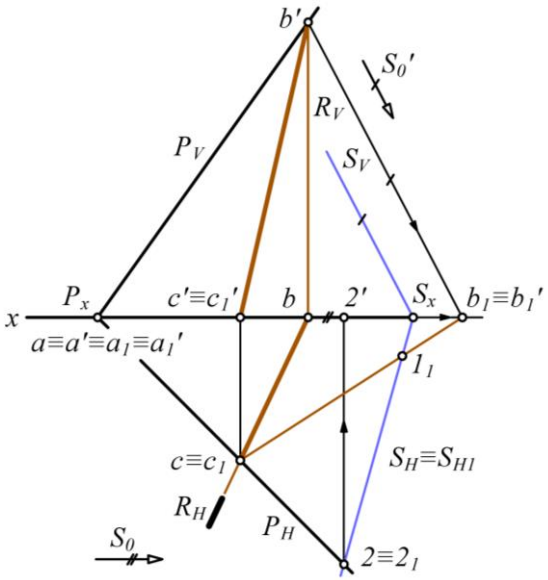


Fig. 13

One point of the straight line intersection of the planes (point ***2***) is defined as the intersection of their horizontal traces P_H and S_H. And to determine the second intersection point (point ***1***), the plane P is intersected by an arbitrary horizontally projecting plane R. The intersection of the planes P and R is the line BC. Projecting this line in the direction of projection, we obtain the line B_1C_1, which, when intersected with the trace S_{H1}, gives point 1_1. The point 1_1 as a result of an inverse transformation gives the second required point ***1*** of straight line of intersection of the planes (Fig. 14).

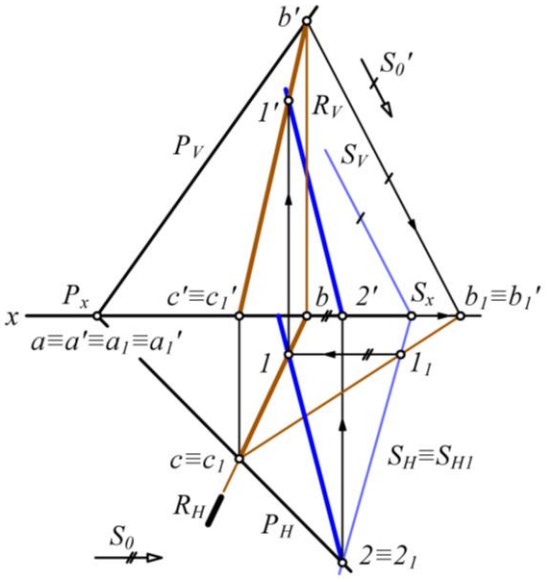


Fig. 14

3.2. PLANES SET BY A TRIANGLE AND TRACES

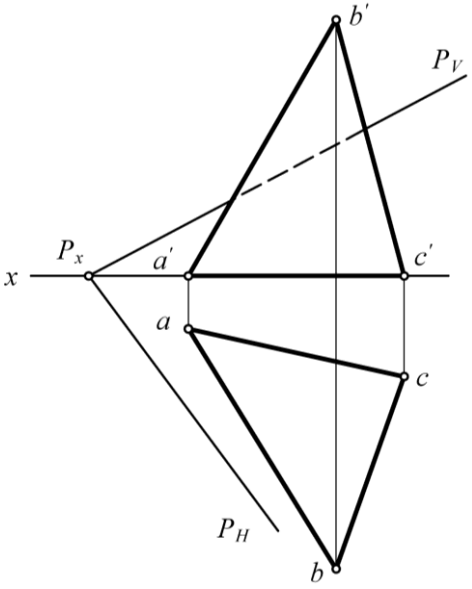


Fig. 15

Let a plane be given in the form of a triangle ***ABC***, the side ***AC*** of which lies in the horizontal plane of projections, and the plane ***P*** is given by the frontal $\boldsymbol{P_V}$ and horizontal $\boldsymbol{P_H}$ traces (Fig. 15).

At first the given planes intersect auxiliary horizontal plane ***S*** (Fig. 16). In the intersection with triangle ***ABC***, we obtain a straight line ***12***, and in the intersection with plane ***P***, we obtain a horizontal line, the horizontal projection of which is parallel to the horizontal trace of plane P_H (in Fig. 16, the blue lines). The horizontal projections of line ***12*** and the horizontal intersect at point ***m***, which belongs to the line of intersection of the planes. The frontal projection of the point ***m'*** lies in the secant plane ***S*** and belongs to the frontal projection of the line of intersection of the planes ***ABC*** and ***P***.

Similarly, another point of straight line intersection is determined, namely point ***n***, by crossing the given planes with the auxiliary horizontal plane ***U*** (Fig. 17). The auxiliary plane intersects the plane ***ABC*** along the line ***34***, and intersects the plane ***P*** horizontally. The horizontal projections of the horizontal and line ***34*** define the position of point ***n*** (its horizontal projection). The line mn is the horizontal projection of the line of intersection of the given planes, and m'n' is the frontal projection.

The part from the intersection line to the base of ***AC*** of the triangle becomes invisible, as does part of the frontal trace of P_V (Fig. 17). Fig. 18 shows the general construction.

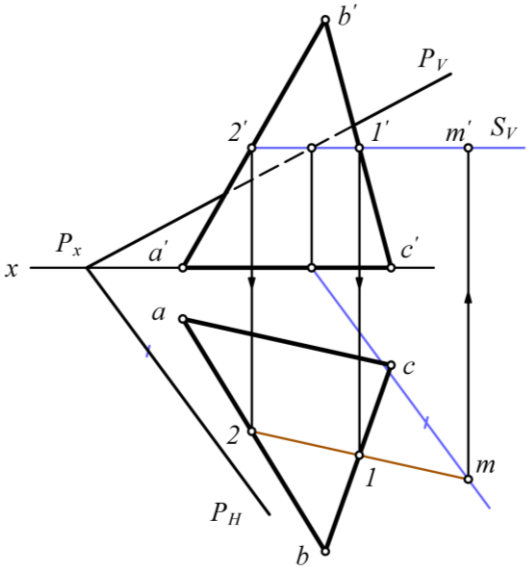
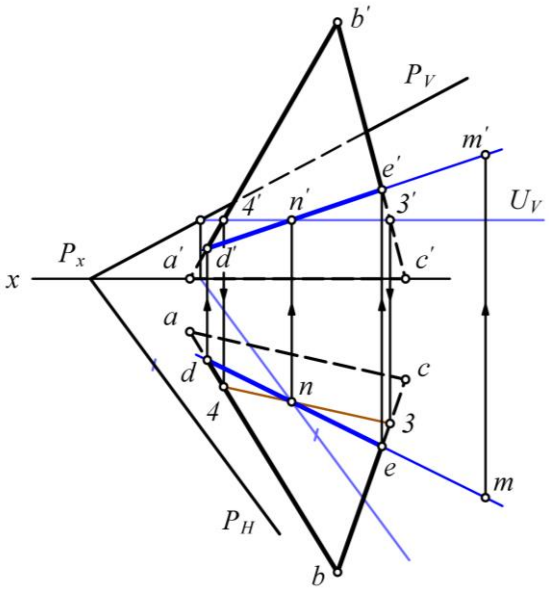


Fig. 16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e
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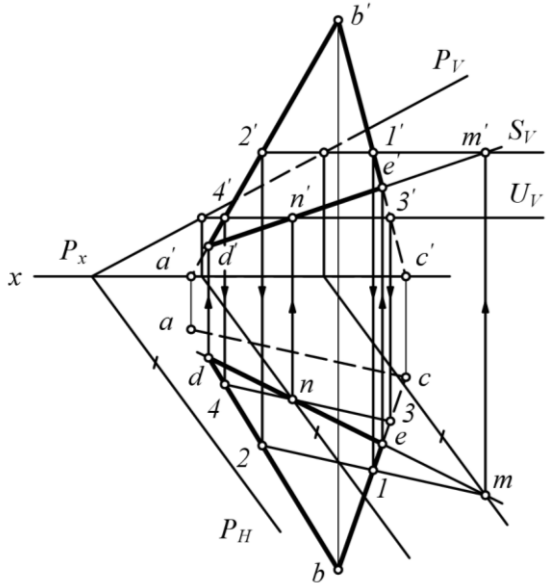
P_H
b

Fig. 17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2
1
m
P_H
b

Fig. 18

In fig. 19 the intersecting frontally projecting ***S*** and ***U*** planes are carried out through the sides of a triangle of ***AB***, ***BC***. These planes intersect the plane ***P*** along lines ***12*** and ***34*** (points ***1*** and ***2*** are the intersections of the corresponding traces of the planes ***P*** and ***U***, and points ***3*** and ***4*** are the intersections of the planes ***P*** and ***S***). The horizontal projections of lines ***12*** and ***34*** at the intersection with the horizontal projections of sides ***bc*** and ***ba*** give points ***e*** and ***d***, which form the horizontal projection of the line of intersection. Their frontal projections belong to the frontal projections of the corresponding sides of the triangle.

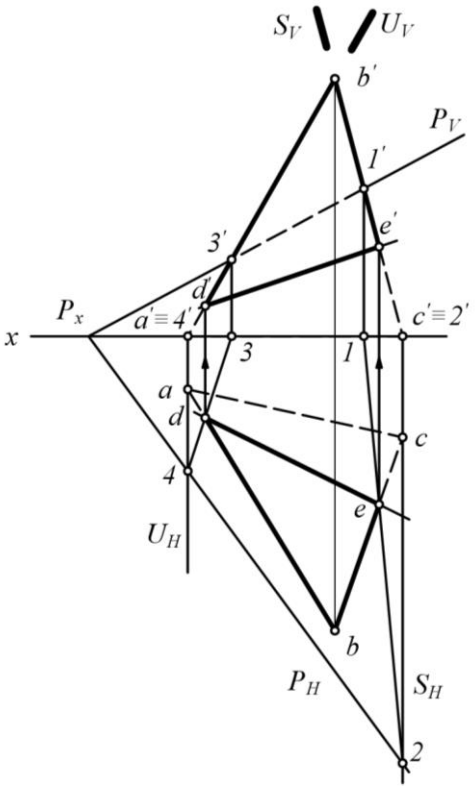

Fig. 19

Using ***projection transformation methods*** to solve the same problem does not differ fundamentally from the case when both planes were given as triangles. The construction is even simplified, since in the

plane P there is no need to construct a horizontal or frontal, since they are already given - these are the horizontal and frontal traces of the plane. Therefore, using ***the method of replacing projection planes***, the new axis x_1 is placed (Fig. 20) perpendicular to the horizontal trace P_H, replacing the frontal plane V with a new frontal plane V_1, in which the Z coordinates of all points don't change.

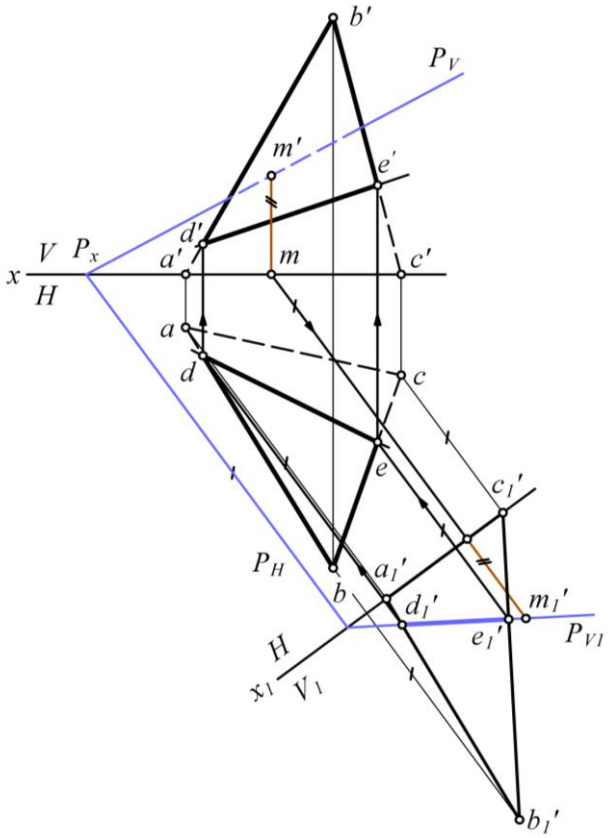

Fig. 20

The new position of the frontal trace P_{V1} is determined using an arbitrary point M, which is set on the frontal trace of the P_V plane (Fig. 20) (the Z coordinate of the point on the frontal planes of the V and V_1 projections is unchanged). As a result of this replacement, the plane P becomes frontally projecting, and its new frontal trace P_{V1} in the

intersection with the new projections of the sides of the triangle gives the points e_1' and d_1', which form the frontal projection of the straight line in the new system of projection planes H and V_1. The resulting straight line of intersection along the lines of projection connection is transferred to the horizontal and frontal projections of triangle ABC.

In fig. 21 the solution of this task is illustrated with ***the method of plane-parallel movement***. The new horizontal trace of the P_{H1} plane is placed perpendicular to the axis x (it is rotated by an angle φ), and the plane becomes frontally projecting. During such movement coordinates Z all points, the sizes of elements of the horizontal projection and their relative situation don't change. The new position of the frontal trace P_{V1} is determined, as in the previous case, by an arbitrary point M, which is set on the frontal trace of the plane. In the new position, the intersection of the planes is determined by the intersection of the trace P_{V1} with the new projections of the sides of the triangle (points e_1' and d_1').

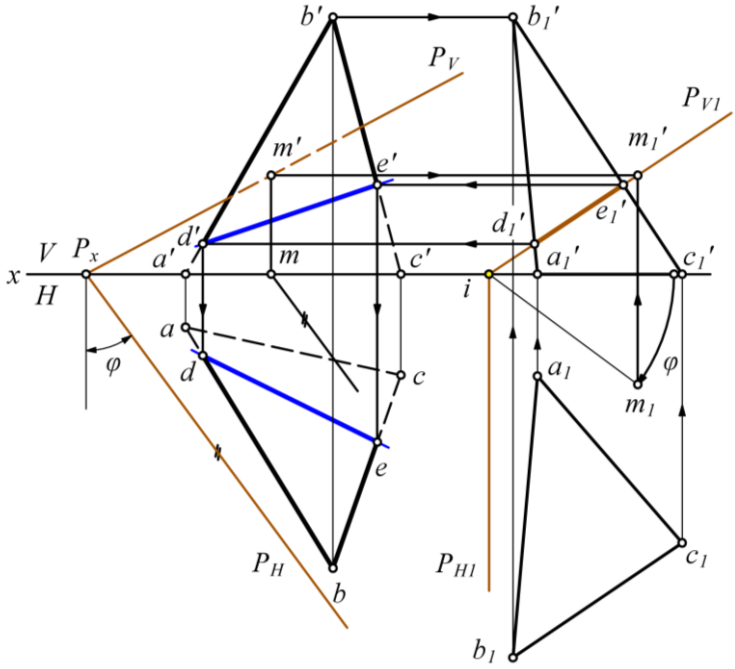

Fig. 21

Fig. 22 illustrates use of ***the method of oblique-angled auxiliary projection***.

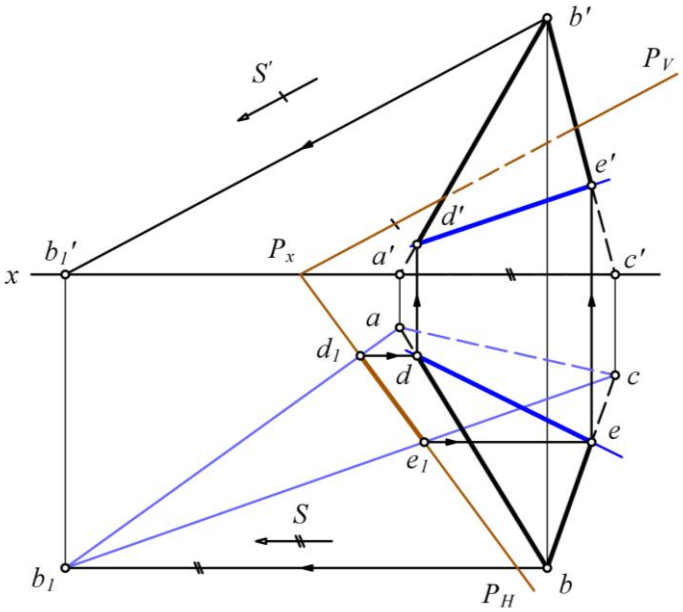

Fig. 22

The direction of projection coincides with the line of the plane ***P***, namely, with the frontal trace of the plane $\boldsymbol{P_V}$ (the horizontal projection of the frontal trace coincides with the axis ***x***). The projection occurs on the horizontal projection plane ***H***. After such a projection, the plane ***P*** turned into a straight line that coincided with the horizontal trace $\boldsymbol{P_H}$, and the triangle turned into a triangle $\boldsymbol{acb_1}$ (side ***ac*** lies in the horizontal plane of projections, so it remained stationary). The intersection of the planes – line $\boldsymbol{d_1e_1}$ – is returned by inverse projection to the horizontal projection of the triangle, and from there along the lines of the projection connection, namely to the frontal projection.

4. QUESTIONS FOR INDEPENDENT TRAINING

1. What is orthogonal projection of a point?

2. How many coordinates of a point are necessary to set the provision of a point in three-dimensional space?

3. How many orthogonal projections of a point set its position in three-dimensional space?

4. What do two points define – a line or a segment?

5. What straight lines of special situation do you know? And planes?

6. What straight lines on their relative position exist?

7. How it is possible to set the plane?

8. What is a trace of a line? A plane? How to construct them?

9. What main lines of the plane do you know? Their properties.

10. How to construct the intersection of two planes?

LIST OF RECOMMENDED LITERATURE

1. Dol’s’kyy YE. YE., Yevstyfeyev M. F. Zbirnyk zadach z narysnoyi heometriyi. Kyyiv : Derzhbudvydav URSR, 1961. 196 s. [in Ukrainian].

Дольський Є. Є., Євстифеєв М. Ф. Збірник задач з нарисної геометрії. Київ : Держбудвидав УРСР, 1961. 196 с.

2. Krivtsov V. V., Puhachov YE. V., Karavan V. V., Makarenko R. M. Inzhenerna hrafika ta osnovy budivel’noho kreslennya: navch. posib. Rivne : NUVHP, 2024. 691 s. [in Ukrainian].

Крівцов В. В., Пугачов Є. В., Караван В. В., Макаренко Р. М. Інженерна графіка та основи будівельного креслення : навч. посіб. Рівне : НУВГП, 2024. 691 с.

3. Narysna heometriya / Mykhaylenko V. YE., Yevstifeyev M. F., Koval’ov S. M., Kashchenko O. V. Kyyiv : Vyshcha shkola, 2004. 303 s. [in Ukrainian].

Нарисна геометрія / Михайленко В. Є., Євстіфеєв М. Ф., Ковальов С. М., Кащенко О. В. Київ : Вища школа, 2004. 303 с.

4. Puhachov YE. V., Zdanevych V. A., Litnits’kyy S. I., Kundrat T. M. Zbirnyk zadach z narysnoyi heometriyi z rozv’yazkamy. Ortohonal’ni proyektsiyi : navchal’nyy posibnyk / Za redaktsiyeyu d.t.n., prof., YE. V. Puhachova. Rivne : Volyn. oberehy, 2021. 316 s. [in Ukrainian].

Пугачов Є. В., Зданевич В. А., Літніцький С. І., Кундрат Т. М. Збірник задач з нарисної геометрії з розв’язками. Ортогональні проєкції : навчальний посібник / За редакцією д.т.н., проф., Є. В. Пугачова. Рівне : Волин. обереги, 2021. 316 с.

5. Mykhaylenko V. E., Naydysh V. M. Tlumachennya terminiv z prykladnoyi heometriyi, inzhenernoyi ta kompмyuternoyi hrafiky. K. : Urozhay, 1998. 197 s. [in Ukrainian].

Михайленко В. Е., Найдиш В. М. Тлумачення термінів з прикладної геометрії, інженерної та комп’ютерної графіки. К. : Урожай, 1998. 197 с.

6. Narysna heometriya ta osnovy arkhitekturnoyi hrafiky : navchal'nyy posibnyk / S. I. Pustyul'ha, V. R. Samostyan. Luts'k : Vezha, 2020. 318 s. [in Ukrainian].

Нарисна геометрія та основи архітектурної графіки : навчальний посібник / С. І. Пустюльга, В. Р. Самостян. Луцьк : Вежа, 2020. 318 с.

7. Narysna heometriya. navch. posib. u 2-kh knyhakh. Knyha I. Ortohonal'ni proektsiyi / S. M. Koval'ov, S. I. Botvinovs'ka, K. M. Hermash ta in. K. : KNUBA, 2018. 166 s. [in Ukrainian].

Нарисна геометрія. навч. посіб. у 2-х книгах. Книга І. Ортогональні проекції / С. М. Ковальов, С. І. Ботвіновська, К.М. Гермаш та ін. К. : КНУБА, 2018. 166 с.

Information resources

1. National Library named after V. I. Vernadskyi. URL: http://www.nbuv.gov.ua/

2. Rivne Regional Universal Scientific Library (Rivne, st. Oleksandra Borysenka, 6). URL: http://www.libr.rv.ua/

3. Scientific library of NUWEE (Rivne, st. Oleksy Novaka, 75). URL: http://nuwm.edu.ua/naukova-biblioteka

4. Digital repository of NUWEE. URL: http://ep3.nuwm.edu.ua/view/types/metods/