PROBLEMS OF COMPUTER MODELING OF NONLINEAR FILTRATION PROCESSES OF UNDERGROUND HYDRODYNAMICS

HladkaO. M., Karpovych I. M.

INTRODUCTION

Today, the problem of satisfying the needs of our country in oil and gas is not only a priority task for ensuring sustainable development of Ukraine, but also serves as a strategic issue for state security. Most of the oil and gas deposits in Ukraine are exhausted and are at the final stage of development, characterized by a decline in wells and a deterioration of hydrocarbon production conditions. At the same time, due to zonal and layer heterogeneity of reservoirs, increased oil viscosity and manifestations of other factors arising in the course of exploitation of deposits, 35-60% of initial oil reserves remain non-extracted, while only about 20% of wells have an actual flow corresponding to the intact (primary) layers, and in the rest – in 2-30 times lower [1]. The reasons for the decrease in productivity are the colmatation of the zone around the well during the opening of the layers and during the exploitation of wells by the particles of drilling and cement mortars, products of corrosion of equipment, sediments formed in the interaction of reservoir water and impregnated solutions, resins and asphaltenes, sand and clay particles of the skeleton of the rock, increase in water content hydrocarbons and formation of stable high viscosity water-oil emulsions, as a result of penetration into the formation of filtrate of technological fluids, etc. [2].

Modern oil and gas extraction specialists develop and actively apply in practice methods for intensifying oil and gas extraction from existing deposits, among which (according to the classification by V. S. Boyko[2]) four main groups of methods of influence on deposits are distinguished: hydrodynamic methods (cyclic flooding, change of direction of filtration flows, creation of high injection pressure or forced removal of a product), physical and chemical methods (application of active

[1] Kondrat O. (2013) Study of methods of hydrocarbon recovery enhancement from the depleted oil fields. *AGH Drilling Oil and Gas Quarterly*. Vol. 30, № 1. 127–145.
[2] Бойко В.С., Бойко Р.В., Грибовський Р.В. та ін. Видобування нафти в ускладнених умовах: Монографія. Івано-Франківськ: Нова Зоря, 2013. 711 с.

impurities), gas methods (oil displacement with high pressure gas), thermal methods (used dipping different kind of coolants).

Under these conditions, an important task is to develop approaches to the assessment and feasibility study of the effectiveness of these methods and to thoroughly study the peculiarities of the complex processes of displacement of hydrocarbons from heterogeneous layers that have undergone deformations during the operation of deposits, which in turn requires the development of a mathematical methodology a description of these processes (taking into account the reciprocal influence of their parameters on the initial characteristics of the medium) with a view to further research using computer simulation.

The geological formation is a very complex object that can not be explored precisely. At the same time, the results obtained using models of underground hydro-gas dynamics should be used in the process of making specific technological decisions. In connection with the lack of information on deposits with a heterogeneous structure of productive layers in the creation of appropriate mathematical models, the deposit (reservoir) is divided into areas (zones) within which the productive stratum can be considered homogeneous with sufficient accuracy or such that its main characteristics are given in a certain way. However, these zones are curvilinear, with complex geometry, which is not completely defined. Simulation by their layered structures is not always justified, as it can lead to significant deviations of calculated results from real data.

Taking into account the current researches on underground hydrodynamics, in particular, the mechanisms of fluid motion and the technogenically induced processes of the borehole deformations of the rock, for the modeling of zonal-inhomogeneous layers the authors introduced the following model objects such as LEF-layers (LEF – abbreviation from Lines of Equipotential and Flow), which determine the nonlinear-layered structures in which the parameters characterizing the main filtration properties of the medium (for example, the coefficient of permeability of the formation) are piecewise constant functions dependent on the quasipotential and the flow function, while the unknown geometry of the zones is determined by the corresponding equipotential lines and flow lines, which are calculated in the process of solving the problem [3]. This approach also allows consider the inverse effect of the process parameters on the output characteristics of the medium.

Many researchers in our time, given the enormous potential of modern computer technology, are trying to describe the processes occurring in oil and gas seams, with the help of detailed spatial models. But the deterministic mathematical models for solving them require the exact geological and filtration characteristics of the research object, which in the case of oil and gas layers is very problematic, which may lead to an inadequate description of displacement processes and significant errors in the calculations. Therefore, at the initial stage of the simulation, it is sufficient to use mathematical models of processes of a smaller spatial dimension (namely, two-dimensional) with certain modifications, which would allow to trace the main effects of the process followed by extrapolation to three-dimensional models. Especially since methods of dissemination to spatial problems of flat filtration are known and detailed in the works of many scientists.

Despite a large number of scientific studies on the problems of mathematical modeling of filtration processes in porous mediums, for today there are many problems concerning the scientific substantiation of measures to improve the design quality and efficiency of oil and gas fields development, to establish the peculiarities of the flow of nonlinear displacement processes taking into account the inverse effect of parameters process on the around well zone of the reservoir, etc., connected, first of all, with the choice of mathematical models, which, on the one hand, are adequate and maximally accurately took into account all the factors of real processes, and on the other hand, would be favorable for solving and computer simulation. Taking into account that physical experiments and natural studies in the oil and gas sector are very complex and expensive, mathematical and computer simulation of these processes is relevant and of great practical importance. No less important is the problem of finding (identifying) reservoir parameters by some known data, which

[3] Бомба А.Я., Гладка О.М., Кузьменко А.П. Обчислювальні технології на основі методів комплексного аналізу та сумарних зображень: [монографія]. Рівне: Ассоль, 2016. 283 с.

leads to complex inverse problems for which, to date, there is no generally accepted universal theory and established methods of solution.

1. Modeling of nonlinear filtration processes by synthesis of numerical methods of complex analysis, summary representations and domain decompositions

In the case of complex stratum configuration, well-developed analytical methods can not usually be used, which has become an impetus for the development of a variety of numerical and numerical-analytical methods, most of which are based on the idea the discretization of a corresponding area, which involves breaking it into blocks of fixed size and approximating the corresponding equations relative to them. However, quite often this partitioning is not optimal, since the characteristics of the filtration flow can vary greatly in some parts of the domain and almost do not change – in others, therefore, one has to use uneven partitioning of the domain into blocks whose dimensions need to be adapted to the solution. In addition, there is a problem with the "adequacy" of the discretization of the corresponding convection equations, which are nonlinear, the solutions of which would not contradict the physical picture of the corresponding process.

An effective method of computer modeling of nonlinear filtration processes of underground hydrodynamics in curvilinear domains, bounded by flow lines and equipotential lines (and, in particular, LEF-layers), is method developed by A. Ya. Bomba and his students [4] [5] [6] on the basis of complex analysis using methods of conformal and quasiconformal mapping that automates the construction of dynamic grids, which are the basis for calculating the value of the field of velocity, the distribution of pressure in the reservoir, the values of total filtration flows and flows between wells, points of stopping of the flow, other characteristics of the modelHowever, the implementation of this approach in many cases is accompanied by considerable difficulties. In particular, it is a problem of choosing a "qualitative"

---

[4] Бомба А.Я., Булавацький В.М., Скопецький В.В. Нелінійні математичні моделі процесів геогідродинаміки. К.: Наукова думка, 2007. 308 с.

[5] Бомба А.Я., Каштан С.С., Пригорницький Д.О., Ярощак С.В. Методи комплексного аналізу: Монографія. Рівне: НУВГП, 2013. 415 с.

[6] Bomba A.Ya., Yaroshchak S.V., Myslyuk M.A. (2013). Mathematic modelling of thermodynamic effects in a gas formation well bore zone, *Journal of Hydrocarbon Power Engineering*, №1, 1–4.

initial approximation of the desired functions, the accumulation of computational errors in the successive iterative recalculations of the values of functions in the nodes of the calculation grid by the values in the neighboring nodes, the problem of determining the completion conditions of the iterative process of calculating the values of functions in internal nodes at fixed values in external, which, in turn, will still be "refined" at the next stage of computation, and so on.

Formation of the correct mathematical model of such processes is a rather difficult task not only due to the heterogeneity of the medium but also due to the problem of determining the boundaries of the heterogeneity zones, the formulation of the corresponding boundary conditions, and especially considering the need to take into account the reciprocal influence of the process characteristics on the filtration properties of the porous medium [7] [8] [9] [10].

The basis of this research is the idea of synthesis of numerical methods of complex analysis, summary representations and domain decomposition for the purpose of mathematical modeling of nonlinear quasi-ideal filtration processes in technogenically-deformabled porous water-oil and gas LEF-layers whose boundaries of zones of heterogeneity are determined by the sought-after lines of the dynamic grid.

In the papers [3] [7] were formulated the nonlinear boundary value problems, in which the conductivity of the medium depends on the field potential (pressure, push) and on the flow function for single-, two-, and multiply connected curvilinear LEF-domains. The development of the appropriate computing technology for their solution was realized on the basis of the synthesis of numerical methods of quasiconformal

---

[7] Бомба А.Я. Кузьменко А.П., Гладка О.М. Синтез числових методів конформних відображень та сумарних зображень при моделюванні ідеальних полів для криволінійних областей. *Вісник Київського нац. ун-ту ім. Т. Шевченка. Серія: фіз.-мат. науки*. 2012. № 2. 87–94.

[8] Bomba A.Ya., Hladka O.M. (2017). Problems of identification of the parameters of quasiideal filtration processes in nonlinear layered porous media, *J. of Math. Sciences*, № 2 (220), 213–225.

[9] Bomba Andrey Ya., Hladka Elena N. (2014). Methods of Complex Analysis of Parameters Identification of Quasiideal Processes in Nonlinear Doubly-layered Porous Pools**,** *Journal of Automation and Information Sciences,* Vol. 46, Is. 11, 50–62.

[10] Hladka O., Bomba A. (2014). The complex analysis method of numerical identification of parameters of quasiideals processes in doubly-connected nonlinear-layered curvilinear domains, *Journal of Mathematics and System Science,* Vol. 4, № 7 (Ser. No. 29), 514–521.

mappings, summary representations for differential equations with discontinuous coefficients [11], and the domain decomposition using the alternating method by Schwartz. At the same time, the approximation of the values f the desired functions in the internal nodes of the settlement grid (coordinates of the internal nodes of the dynamic grid) in the process of iterations was calculated by the formulas of summary representations method.

In this paper we considering the problems of stationary filtration of finding the harmonic function (potential) $\varphi = \varphi(x, y)$ in the curvilinear LEF-domains G_z ($z = x + iy$): a) ***single-connected***, bounded equipotentials $L_* = \{z : f_1(x, y) = 0\}$, $L^* = \{z : f_3(x, y) = 0\}$ and flow lines $L_0 = \{z : f_4(x, y) = 0\}$, $L^0 = \{z : f_2(x, y) = 0\}$, in which the potential of the field satisfies the boundary conditions $\varphi\big|_{L_*} = \varphi_*$, $\varphi\big|_{L^*} = \varphi^*$, $\left.\frac{\partial \varphi}{\partial \mathrm{n}}\right|_{L_0} = \left.\frac{\partial \varphi}{\partial \mathrm{n}}\right|_{L^0} = 0$, ($-\infty < \varphi_* < \varphi^* < +\infty$ are constants, n is external normal to the corresponding curve); b) ***double-connected***, bounded by two smooth closed contours $L_* = \{z : f_*(x, y) = 0\}$ is internal and $L^* = \{z : f^*(x, y) = 0\}$ is external, in which, for the formation of interconnected domain $G_z^{\Gamma} = G_z \setminus \Gamma$, a conditional incision Γ is made along a certain sought-after line of current (then, L_0 and L^0 are the boundary lines of the domain G_z^{Γ} that are respectively the upper and lower banks section Γ); c) ***triple-connected***, bounded by two internal contours of wells – equipotential lines $L_* = \{z : f_*(x, y) = 0\}$ and $L^* = \{z : f^*(x, y) = 0\}$ and an impenetrable outer contour $L = \{z : f(x, y) = 0\}$, in which two conventional cuts Γ_* and Γ^* are made along such lines of flow (which are lines of separation of flow), which are uniquely determined by the points of the "stoppage" of the flow $H_* \in L$, $H^* \in L$ and $G_z^{\Gamma} = G_z \setminus \left(\Gamma_* \bigcup \Gamma^*\right)$; d) ***with a free boundary*** (depression curve), which is given an additional condition: $\varphi\big|_{BC} = g(y)$. $H \geq y \geq y_* = f^*(x_*)$ ($g(y)$ is some known

[11] Ляшко И.И., Великоиваненко И.М. Численно-аналитическое решение краевых задач теории фильтрации. К.: Наукова думка, 1973. 264 с.

monotonically declining function). The filtration process is described by Darcy's law $\vec{\upsilon} = \kappa \cdot \operatorname{grad} \varphi$ and the equation of continuity $\operatorname{div} \vec{\upsilon} = 0$, where $\vec{\upsilon} = \upsilon_x(x,y) + \mathrm{i}\,\upsilon_y(x,y)$ is the filtration rate, κ is some function, limited and continuous in G_z (or tensor of functions of the second kind), which characterizes the conductivity of the medium and its predisposition to deformation.

By introducing the flow function $\psi = \psi(x,y)$, complexly conjugate to $\varphi = \varphi(x,y)$, we arrive at a more general problem on the quasi-conformal mapping $\omega = \omega(z) = \varphi(x,y) + i\psi(x,y)$ of a given physical domain G_z to the corresponding domain of complex potential $G_\omega = \{\omega = \varphi + i\psi:\ \varphi_* < \varphi < \varphi^*,\ 0 < \psi < Q\}$ with an unknown parameter (total filtration flow) Q:

$$\begin{cases} \dfrac{\partial \varphi}{\partial x} = \dfrac{\partial \psi}{\partial y}, \quad \dfrac{\partial \varphi}{\partial y} = -\dfrac{\partial \psi}{\partial x}, \quad (x,y) \in G_z, \\ \varphi\big|_{L_*} = \varphi_*, \quad \varphi\big|_{L^*} = \varphi^*, \quad \psi\big|_{L_0} = 0, \quad \psi\big|_{L^0} = Q, \\ Q = \displaystyle\int_{L_*} -\frac{\partial \varphi}{\partial y}dx + \frac{\partial \varphi}{\partial x}dy\,. \end{cases} \tag{1}$$

The task (1) (implying the methodology developed in [3]) we replaced by the inverse to it, since, firstly, the domain of the complex potential G_ω is a rectangle (or the union of "glued" rectangles), in contrast to the geometrically complex physical domain G_z, and secondly, the transition to the inverse map automatically solves the problem of the discretization of task for the application of numerical methods, allows us to use the advantages of the method of summary representations for solving the corresponding difference problems, build a dynamic grid of substance remove, determine the total filtration flow Q (without solving the integral equation), etc.

Using the formulas for the transition: $\dfrac{\partial \varphi}{\partial x} = \dfrac{1}{J}\dfrac{\partial y}{\partial \psi}$, $\dfrac{\partial \varphi}{\partial y} = -\dfrac{1}{J}\dfrac{\partial x}{\partial \psi}$, $\dfrac{\partial \psi}{\partial x} = -\dfrac{1}{J}\dfrac{\partial y}{\partial \varphi}$, $\dfrac{\partial \psi}{\partial y} = \dfrac{1}{J}\dfrac{\partial x}{\partial \varphi}$, $J = \dfrac{\partial x}{\partial \varphi}\dfrac{\partial y}{\partial \psi} - \dfrac{\partial x}{\partial \psi}\dfrac{\partial y}{\partial \varphi}$, the corresponding boundary value problem on the inverse conformal (quasi-conformal) mapping

$z = z(\omega) = x(\varphi,\psi) + iy(\varphi,\psi)$ of the domain G_ω at G_z with unknown value of the parameter $Q = \int_0^Q \frac{1}{J}\left(\left(\frac{\partial x}{\partial \psi}\right)^2 + \left(\frac{\partial y}{\partial \psi}\right)^2\right) d\psi$ supplemented by the conditions of the orthogonality of the flow lines and the equipotential lines to the corresponding areas of the boundary of the physical domain, analogously to [3], we reduced to solving in second-order elliptic differential equations:

$$\frac{\partial}{\partial \varphi}\left(\frac{1}{\kappa}\frac{\partial x}{\partial \varphi}\right) + \frac{\partial}{\partial \psi}\left(\kappa \frac{\partial x}{\partial \psi}\right) = 0, \quad \frac{\partial}{\partial \varphi}\left(\frac{1}{\kappa}\frac{\partial y}{\partial \varphi}\right) + \frac{\partial}{\partial \psi}\left(\kappa \frac{\partial y}{\partial \psi}\right) = 0, \quad (\varphi,\psi) \in G_\omega \qquad (2)$$

(it is Laplace equations at $\kappa = 1$), which satisfy the corresponding boundary conditions and conditions for the orthogonality of the lines of the dynamic grid to the corresponding areas of the boundary of the domain [3].

For the numerical solution of the problem, its discretization is performed by replacing the equations (2) with the corresponding finite-difference analogues, defining it in G_ω the orthogonal grid $G_\omega^\gamma = \{(\varphi_i, \psi_j): \varphi_i = \varphi_* + \Delta_\varphi \cdot i,\ i = \overline{0, m+1};$ $\psi_j = \Delta_\psi \cdot j,\ j = \overline{0, n+1};\ \Delta_\varphi = \frac{\varphi^* - \varphi_*}{m+1},\ \ \Delta_\psi = \frac{Q}{n+1},\ m,\ n \in \mathbf{N}\}$, where $\gamma = \Delta_\varphi / \Delta_\psi$ is the conformal invariant. In this case, the boundary conditions and conditions of orthogonality in the vicinity of the boundary of the domain are approximated by the corresponding special numerical-analytic difference equations [3].

The formula for the approximate finding of a conformal invariant γ is obtained on the basis of the condition of "conformal similarity in a small" of the corresponding elementary quadrilateral domains G_ω^γ (the difference grid) and G_z^γ (the dynamic grid formed in the physical domain by intersection of the flow lines and equipotents) [3].

The algorithm for numerical solution of the problem is constructed on the basis of ideas of the method of block iteration by the stepwise parametrization of the magnitude of the conformal invariant of reflection, the required coordinates of the boundary and internal nodes of the dynamic grid. As a result of solving the problem, we found the value of the total filtration flow Q and the values of the desired functions x and y (coordinates of the nodes of constructed in the field of the filtering of the dynamic grid).

At the end of the each iteration, we check the implementation of the conditions for stabilizing the coordinates of the boundary nodes of the dynamic grid, and evaluate the degree of conformance of the received reflection of the complex potential domain to the physical domain by calculating the nonconformities of the Cauchy-Riemann conditions approximation.

The constructed dynamic grids are the basis for the study of other quantitative characteristics, in particular, the velocity of motion $\vec{\upsilon} = \upsilon_x(x, y) + \mathrm{i}\,\upsilon_y(x, y)$, which is found by the formula $\vec{\upsilon} = \overline{\dfrac{d\omega}{dz}} = 1 \Big/ \overline{\dfrac{dz}{d\omega}}$ [3].

In the computational realization of the process of finding solutions of the corresponding boundary value problems, there is a problem of solving systems of linear algebraic equations with poorly determined matrices of large dimension. One of the ways to avoid this problem is to use as its components the numerical-analytical methods of summary representations that were developed by G. M. Polozhii, I. I. Lyashko, A. A. Gluschenko, and others [11] [12].

Methods of summary representations (another name – the methods of P-transformations) are discrete analogues of integral representations, and when applied to the solution of boundary value problems of mathematical physics in a discrete formulation, generalizes classical methods of the theory of potential, integral equations, Green's function, integral transformations and separation of variables. In this case, solutions of tasks are obtained in a closed form as formulas of the summary representations. Depending on the boundary conditions, they are either explicit, or contain a certain number of unknown parameters that are determined from the corresponding system of linear algebraic equations. The method allows you to make a selective account. This is a very important aspect of the technology because that for most technical tasks it is sufficient to know the solution only in those critical nodes

[12] Polozhii G. N. (1965). The method of summary representations for numerical solution of problems of mathematical physics. Pergamon Press, London.

(areas) of the domain that determine the main characteristics of the model. When solving problems using the method of summary representations, most of the unknowns that are part of the finite-difference problem do not directly participate in the direct account, which leads to a decrease in the amount of computational work, and, therefore, makes it possible to avoid accumulation of computational errors. Analytical form of the solution allows not only to find its numerical values, but also to perform some qualitative research without, in fact, additional calculations. By the method of summary representations, problems can be solved (in particular, the theory of filtration, the theory of elasticity, the theory of plates and shells, etc.) in a very large and even unlimited number of nodes of a grid for single- and multi-connected flat and spatial domains. In this case, if for canonical domains (rectangles, strips, and semi-bands, circular regions) and their associations and spatial analogues, the formulas of the summary representations were substantiated by the method's developers, then obtaining the formulas of the summary representations for solving problems for non-homogeneous media in the curvilinear complex configuration areas is essentially an uneasy task, which is primarily due to the problem of finding eigenvalues and eigenfunctions of the corresponding boundary value problems.

The application of the method of summary representations in the computational technologies presented here is also grounded in the fact that it is sufficiently convenient for computer realization. Therefore, probably, now the method and has its "second breath", which is connected, in particular, with the rapid growth of resources of modern computer technology. In addition, the method of summary representations allows a natural way to parallelize the computational process, which could not have been fully exploited by its developers, and which is very relevant nowadays due to the modern development of computer technologies. This is also important because to the developers of the methodology for simulating complex nonlinear filtration processes based on the methods of quasi-conformal mappings A. Ya. Bomba, his students and, in particular, the authors of the work, have repeatedly noticed a significant computational complexity of the presented mathematical models and algorithms for solving the corresponding problems.

2. Modeling of multiply-connected curvilinear LEF-layers by synthesis of numerical methods of complex analysis, summary representations and domain decomposition

Investigation of filtration processes in water-oil and gas technogenically-deformable layers with the help of computer modeling requires the solving of boundary value problems that describe complex nonlinear mathematical models of underground hydrodynamics, which, in turn, requires the development of new information technologies for the realization of such computational processes. One of the ways to improve the approaches used previously is to parallelize the computing process and create multi-threaded computer applications. This requires making changes to the methodology of mathematical modeling of the corresponding processes and modification of previously created algorithms.

In cases of multiply-connected curvilinear LEF-layers that simulate the interaction of several injection and operational wells in the productive layer, the complexity of the application of such a technique consists in incomplete certainty of the form of the domain of complex quasipotential , which depends on the influence of many factors: the configuration of the physical domain, in particular, the mutual placement wells, methods of conducting conditional cuts to transform a multiply-connected domain into a single-connected one, the relationship between the values of the boundary potentials, etc.

Note that in the works of school A. Ya. Bomba's [5] an approach to modeling filtration processes using mathematical tools for complex analysis for triple-connected domains bounded equipotential lines (two wells and external contour of power supply) and fourth-connected domains bounded equipotential lines (three horizontal wells in layers) and lines of flow, which are determined by the desired points of the "stopping" on the external impenetrable contour, was proposed. In these works, on the basis of heuristic considerations with the following logical justification, possible cases of formation of the flow were established, depending on the ratio of the values of the boundary potential, and the procedures of the automated selection of the corresponding case were developed. The algorithms of solving such problems, which allow to construct dynamic grids, find lines of the current section and calculate the values of velocity and value of different types of flows, are proposed for separate intermediate and key cases. However, this approach necessitates the need for each case (for triple-connected domains 9 such cases are considered, for fourth-connected domains – 23, etc.) separately build an algorithm for solving a problem, having previously performed the so-called "algorithm of case selection" which consists in solving two auxiliary subtasks.

In this paper, we propose a slightly different approach to the classification of cases of flow formation, which makes it possible to unify for all cases the problem of the inversion of quasiconformal mappings, their difference analogs and solution algorithms, as well as the combination of methods of quasi-conformal mappings with numerically-analytic methods of summary representations. In spite of the ambiguity of the quasiconformal mappings of the multiply-connected LEF-domains, in most cases the formation of the current domain of the complex quasipotential by special conduction of conditional cuts of the physical domain can be reduced to a polygon whose sides are parallel to the axes of coordinates and which is considered as a set in a certain way "glued" between are rectangles.

We consider the stationary displacement process described by the equations $\vec{\upsilon} = \kappa_f \cdot \operatorname{grad} \varphi$ and $\operatorname{div} \vec{\upsilon} = 0$, in the curvilinear LEF-domain G_z, which is triple-connected, bounded by closed equipotential lines – two internal contours of wells L_1, L_2 and an external contour of power supply L_3 (case T), or fourth-connected, bounded by three equipotential lines (contours of wells) $L_s = \{z : f_s(x, y) = 0\}$ ($s = 1, 2, 3$) and impenetrable contour $L = \{z : f(x, y) = 0\}$ (case F), with the corresponding boundary potentials given on the equipotentials: $\varphi|_{L_s} = \Phi_s$ ($s = 1, 2, 3$).

If in the preceding paragraphs the corresponding domain of the complex quasipotential was a rectangle with an unknown height and was uniquely determined by the value of the total flow (the flow through the corresponding equipotential line)

$Q = \int_{L_*} -\upsilon_y dx + \upsilon_x dy$, then for a given curvilinear domain with three equipotentials the corresponding domain of the complex quasipotential acquires a different geometric configuration depending on the relation between the boundary potentials $\Phi_s, s = \overline{1,3}$. However, in all these cases there is a quasi-conformal mapping in which the corresponding domain of the complex quasipotential is a polygon, "composed" of rectangles, the set of variants of the formation of the flow is similar, and the unambiguous choice of such a variant (and hence the uniqueness of the corresponding quasi-conformal map) is determined by the values of flows through the equipotentials L_s: $Q_s = \int_{L_s} -\upsilon_y dx + \upsilon_x dy$ ($s = \overline{1,3}$) and the unknown potential φ_H of the point H through which the flow lines pass (case T), and additionally the potentials $\varphi_{H_*}, \varphi_{H^*}$ of the points of the "stopping" of the flow H_* and H^* on the contour L (case F).

By making the conditional cut $\Gamma = \bigcup_{i=1}^{3} \Gamma_i$ along the unknown lines of the section of the current passing through the required point H (case T) or the points H_*, H^* (case F), and obtaining a single-connected domain $G_z^\Gamma = G_z \setminus \Gamma$ and introducing, in the same way as [3], the flow function $\psi = \psi(x, y)$, quasiconformally conjugate to φ, we arrive at the problem on the quasi-conformal mapping $\omega = \omega(z) = \varphi(x, y) + i\psi(x, y)$ domain G_z^Γ to the corresponding domain of complex quasipotential. In this case, the form of domain of the complex quasipotential is different for different cases of formation of the flow, which depend on the relations between the boundary potentials. However, if the corresponding domain of the complex quasipotential is presented as the union of four adjacent rectangles, then all cases of the formation of flow are reduced to three situations that fundamentally differ from each other, and without there being any restrictions on the correlation between the boundary potentials [13].

---

[13] Гладка О.М. Системний підхід до математичного моделювання фільтраційних процесів у багатозв'язних криволінійних LEF-пластах. *Системні дослідження та інформаційні технології*. 2016. № 2. С. 58–73.

Denote: $\varphi_* = \min_s \Phi_s$, $\varphi^* = \max_s \Phi_s$ $(s = \overline{1,3})$, $\varphi_0 = \sum_{s=1}^{3} \Phi_s - (\varphi_* + \varphi^*)$; L_*, L^*, L_0 are the contours corresponding to these potentials L_s; Q_*^*, Q_*^0, Q_0^* are the magnitudes of flows Q_s through the contours L_*, L^*, L_0. Then we have three different configurations of the complex quasipotential domain, which correspond to the following cases: 1) $Q_*^0 = 0$, $Q_*^* > 0$, $Q_0^* > 0$ – there is no flow between the contours L_*, L_0, 2) $Q_0^* = 0$, $Q_*^0 > 0$, $Q_*^* > 0$ – there is no flow between the contours L^*, L_0, 3) $Q_*^0 > 0$, $Q_*^* > 0$, $Q_0^* > 0$ – there is the presence of flows between all contours. At the same time, the uniqueness of the solution of the corresponding problems is provided by the definition of three unknown parameters: two values of flows and the value of the potential φ_H of the point of the flow section (for cases 1 and 3) or three values of flows (for case 2) for the triple-connected domain and definition in addition to two other values of the potentials $\varphi_{H_*}, \varphi_{H^*}$ of points of the "stopping" of the flow H_* and H^* on the contour L for the fourth-connected domain.

Such an approach to the classification of cases for forming the flow and assigning the domain of complex potential is substantially different from that in [5], and allows us to unify the formulation of problems, their difference analogs and solving algorithms.

In general, the complex quasipotential domain has the form: $G_\omega = \bigcup_{k=1}^{4} G_\omega^k$, $G_\omega^k = \{\omega = \varphi + i\psi : \varphi_*^{(k)} \le \varphi \le \varphi^{*(k)}, 0 \le \psi \le Q^{(k)}\}$,

where $\varphi^{*(1)} = \varphi_*^{(2)} = \varphi^{*(3)} = \varphi_*^{(4)} = \varphi_H$, $\varphi_*^{(3)} = \varphi_*$, $\varphi^{*(4)} = \varphi^*$, $\varphi_*^{(1)} = \begin{cases} \varphi_*, & Q_*^0 \ne 0, \\ \varphi_0, & Q_*^0 = 0, \end{cases}$

$\varphi^{*(2)} = \begin{cases} \varphi^*, & Q_0^* \ne 0, \\ \varphi_0, & Q_0^* = 0, \end{cases}$ $\quad Q^{(1)} = \begin{cases} Q_*^0, & Q_*^0 \ne 0, \\ Q_0^*, & Q_*^0 = 0, \end{cases}$ $\quad Q^{(2)} = \begin{cases} Q_0^*, & Q_0^* \ne 0, \\ Q_*^0, & Q_0^* = 0, \end{cases}$

$Q^{(3)} = Q^{(4)} = Q_*^*$.

Such a description of the domain of complex quasipotential is, generally speaking, undetermined, since the values of flows Q_*^*, Q_*^0, Q_0^* are unknown, but, given the monotonically-increasing dependence Q_s on potentials Φ_s, and knowing the boundary potentials, we can assume, in most cases, that the ratio of the flows between the contours to the numerical solution of problem will be assumed.

The corresponding problems in the sub-domain G_ω^k ($k=\overline{1,4}$) on the inverse quasiconformal mapping of the domain G_ω on G_z^Γ are reduced to problems of type [13] with the corresponding boundary conditions, conditions of the orthogonality of the lines of the dynamic grid to the boundary, and the conditions of "split" and periodicity on the sections.

In the domain G_ω we define an orthogonal grid $G_\omega^\gamma = \{(\varphi_i, \psi_j)\}$, where

$$\varphi_i = \begin{cases} \varphi_*^{(1)} + i\cdot\Delta_{\varphi 1},\ i=\overline{0,m_1}, \\ \varphi_*^{(2)} + (i-m_1)\cdot\Delta_{\varphi 2},\ i=\overline{m_1+1, m+1}, \\ \varphi_*^{(3)} + i\cdot\Delta_{\varphi 3},\ i=\overline{0,m_3}, \\ \varphi_*^{(4)} + (i-m_2)\cdot\Delta_{\varphi 4},\ i=\overline{m_3+1, m+1}, \end{cases} \qquad \psi_j = \begin{cases} j\cdot\Delta_{\psi 1},\ j=\overline{0,n_1+1}, \\ j\cdot\Delta_{\psi 2},\ j=\overline{0,n_2+1}, \quad m_2 = m - m_1, \\ j\cdot\Delta_{\psi 3},\ j=\overline{0,n_3+1}, \quad m_4 = m - m_3, \\ j\cdot\Delta_{\psi 4},\ j=\overline{0,n_4+1}, \end{cases}$$

$$\Delta_{\varphi k} = \frac{\varphi^{*(k)} - \varphi_*^{(k)}}{m_k+1}, \qquad 2n = n_1+n_2+n_3+n_4, \qquad \Delta_{\psi k} = \frac{Q^{(k)}}{n_k+1}, \qquad \gamma_k = \frac{\Delta_{\varphi k}}{\Delta_{\psi k}},$$

$m, m_1, m_3, n, n_1, n_2, n_3 \in \mathbf{N}$, $k=\overline{1,4}$.

We find the value of the desired functions in the internal nodes of the calculated grid of domain of the complex quasipotential (a set of adjacent along the vertical and horizontal lines of the grid rectangles), taking into account the conjugation conditions, by combining the methods of summary representations for differential equations with discontinuous coefficients with the alternating method by Schwartz of domain decomposition (see, for example, [14]).

The application of the Schwarz alternating method for the decomposition of the layered domain on the layers of constancy of the conductivity (in the regions with "overlays") makes it possible to effectively find continuous solutions to problems with discontinuous coefficients, to solve problems in more "convenient" sub-

[14] Василевский Ю.В., Ольшанский М.А. Краткий курс по многосеточным методам и методам декомпозиции области. М: МГУ, 2007. 105 с.

domains, rather than the whole field of the original problem, to parallelize the computational process, since calculations in the sub-domains at each iteration step are independent of each other and are executed in parallel with the use of modern computer technologies.

3. Identification of parameters of nonlinear filtration processes in LEF-layers by synthesis of numerical methods of complex analysis, summary representations and domain decompositions

An important problem in the simulation of complex filtration processes, in particular in oil and gas reservoirs, is the development of methods for evaluating the basic parameters of these processes. Despite the large number of scientific studies in recent years in this direction, namely, the methods based on the Tikhonov's principle of regularization [15] [16] [17], the gradient methods aimed at the identification of the parameters of multicomponent distributed systems, reduced to the construction of gradients of the quadratic residual functionals according to the solutions of direct and inverse problems, and described in the works by Deineka and Sergienko (see, e.g.,[18]), the methods of boundary integral equations [19], etc, however, some special classes of tasks for identifying the parameters of filtration processes, which are described by elliptic systems of differential equations, were left out of the attention of the researchers.

The constructive approach proposed by the authors [3] to the simulation of filtration processes for curvilinear layered LEF-layers, bounded by lines of flow and equipotential lines, consisting of a combination of methods of quasi-conformal mappings and summary representations with decomposition of the problem on the layers of the stability of the permeability coefficient can be successfully used not only for studying and numerical solving of direct problems for the construction of a

---

[15] Тихонов А.Н., Арсенин В.Я. Методы решения некорректных задач. М.: Наука, 1986. 288 с.
[16] Vogel C.R. (2002) Computational methods for inverse problems. Philadelphia: SIAM. 183 p.
[17] Kabanikhin S.I. (1995) Numerical analysis of inverse problems. *Journal of Inverse Ill-posed Problems*. 3, No. 4. 278–304.
[18] Сергиенко И.В., Дейнека В.С. Идентификация параметров системы конвективно-диффузионного переноса. *Кибернетика и системный анализ*, 2009. № 1. 42–63.
[19] Pomp A. (1998) The boundary-domain integral method for elliptic system. With application in shells. *Lecture Notes in Mathematics*. Vol. 1683. Berlin, Heidelberg: Springer-Verlag. 163 p.

dynamic grid, finding the velocity field, total flows, and other characteristic parameters of the model, but also for numerical finding of solving of a certain class of inverse problems [8 9 10].

In the given work a method of numerical solving of nonlinear model problems of identification of values of the piecewise constant coefficient of permeability in the subdomains of constancy, taking into account the inverse influence on the configuration of these subdomains of the potential of the field and of the flow function, as well as the characteristic values the potential on the equipotential lines of the section of these subdomains and the values of local flows through the boundaries, are limited by lines of flow, separating these subdomains in curvilinear nonlinear double-layered domains was developed on the basis of synthesis of numerical methods of complex analysis and summary representations. Here, under nonlinearly double-layered terms, we mean the medium with a piecewise constant coefficient of permeability, the domain of stability of which is determined by the corresponding sought-after equvopotential lines and lines of flow. This type of problem arises, in particular, in modeling the reverse effect of the process of displacement of hydrocarbons from non-uniform oil and gas deposits on the initial characteristics of the medium. Thus, in some zones of the breed pores can be clogged with mechanical impurities, paraffins or resinous substances, the process of formation and accumulation of sediment of suspended particles (colmatation) passes, which leads to a decrease in the coefficient of permeability and porosity of the medium. As a rule, there are many cracks in the around wells zones, the size of which depends to a large extent on the operation of wells and significantly affects the permeability of the formation. In addition, the change in filtration characteristics of the rock may occur in the around wells zones due to the displacement (washing out) of its small particles (suffusion), which also causes changes in the permeability of the medium.

In the work, the above-mentioned method is used to identify the characteristic parameters of the process of stationary isothermal filtration of the non-compressible liquid (oil) in a horizontal non-uniform element of the oil reservoir under the rigid water pressure regime, which contains one operating exploitative well and several randomly located non-operating wells, from which the necessary data are obtained.

Suppose that at the design stage and the initial stage of the development of the deposit a probabilistic law for the distribution of the permeability of the productive layer was established, which, in the course of exploitation of the deposit in connection with the new data received and through changes in the filtration characteristics of the rock needs to be clarified.

Parameters of the filtration flow in an element of the oil reservoir are calculated using the method of P-transformations (summary representations), which has a number of advantages. When solving problems using methods of summary representations, most unknowns included in the difference task do not take part in the direct account, which provides a reduction in the amount of computational work. The solution of the problem is obtained in a complete analytical form – in the form of so-called formulas of summary representations, most of which data are explicitly calculated and only a relatively small number of unknown parameters are determined in accordance with generated systems of linear algebraic equations. Formulas of summary representations provide the possibility of selective computations in certain fragments of the calculated domain, which allows, in particular, to avoid accumulation of computational errors. These methods are stable and well adapted to computer realization, and the analytical form of the solution allows also to carry out some qualitative research without finding numerical values.

Thus, a computational technology was developed, in which the methods of summary representations were used as a component of computational procedures developed in [5], resulting in significant improvements. Since, in the iterative approximations of the internal nodes of the dynamic grid, only the surrounding nodes were used in [5], resulting in the desired functions during the iterations, although they became increasingly "harmonious", but not complex-conjugate. The use of methods of summary images made it possible in the complex (in total) at each iterative step to take into account the influence of the boundary and surrounding internal nodes, which accelerated the achievement of the conjugation of the desired harmonic functions. This technology is successfully used both for solving direct filtration problems and for finding solutions of a certain class of inverses.

It was proposed a constructive approach to the solution of the nonlinear model filtration problems of the theory of complex quasipotential for nonlinear layered curvilinear domains under the conditions of identification of the parameters: the conductivity coefficient and the characteristic values of the potential on the equipotential lines of separation of the layers. The application of synthesis of the numerical methods of complex analysis, summary representations, and domain decomposition by the Schwartz method for this class of problems enables us to significantly accelerate the attainment of conjugation of the corresponding required harmonic functions and, to a large extent, avoid the accumulation of calculation errors; moreover, it is convenient for the computer realization.

For the computer realization of the presented problem, the previously developed algorithm of the solution was modified by using multithreading in the module (procedure) for calculating the coordinates of the nodes of the dynamic grid by the method of summary representations. Unlike most other network difference methods, in which the calculation of the value of the function requires the use of values in all adjacent nodes, the methods of aggregate images allow you to calculate the values in all nodes on the same line of the grid simultaneously and independently of each other. In the test program was organized four threads (a computer with a quad-core processor was used), whose functions were the formulas of summary representations, and which in the cycle simultaneously counted values at all nodes of the same line, stopping at the barrier to synchronize the process when moving to the next line. The use of multithreading significantly reduced the execution time of the program and made it possible to more efficiently use the resources of the computer.

CONCLUSIONS

The computational technology and complex of applications that implement appropriate algorithms for solving nonlinear boundary value problems, in which the coefficient of conductivity of the medium depends from the potential of field and from the function of flow, for one-, two- and multiply-connected curvilinear LEF-domains bounded by lines flow and equipotential lines, using summary representations methods for differential equations with discontinuous coefficients (in the cases of layered environments), or constructed numerical-analytic representations of solutions (which generalize summary representations methods in cases heterogeneous environments), were created. The methodology a combination of numerical methods quasiconformal mappings with the task decomposition using of alternating method by Schwarz for separating the complex quasipotential domain into subdomains with “overlays” was developed. The approach and the algorithms numerical identificatoin of parameters quasiideals processes in nonlinear-layered and nonlinear doubly-layered LEF- layers were proposed.

SUMMARY

The paper is devoted to the problems of the mathematical and computer modeling of nonlinear, quasiideals, filtration processes in technogenic-deformable reservoirs of water or oil and gas, in which geometry of zones of heterogeneity determined considering the reverse influence characteristics of the process on the conductivity of environment, and to the development of the methods for solving appropriate boundary value problems on based the synthesis of numerical methods of complex analysis, summary representations and task decomposition with the ability to determine the model parameters. The problem of ambiguity construction the domain of the complex quasi-potential for multiply-connected LEF-domains that model the interaction of injection and production wells in oil and gas reservoirs was solved. It was proposed such a classification cases of forming flow, allowing unify the formulation of problems of inversion of quasiconformal mappings and their difference analogues. The computational technology for solving the problem, that automatically constructs the dynamic grids, finds unknown separation lines and point "suspension" of the flow, calculates the total flow and so on was constructed. Developed algorithms automatically solves the problem of choice of units and the construction of a dynamic grid, finding the lines between the layers constant coefficient of conductivity of the medium, calculate the total flow and calculate the field values of speed, other characteristics of the model.

REFERENCES

1. Kondrat O. (2013) Study of methods of hydrocarbon recovery enhancement from the depleted oil fields. *AGH Drilling Oil and Gas Quarterly*. Vol. 30, № 1. 127–145.

2. Бойко В.С., Бойко Р.В., Грибовський Р.В. та ін. Видобування нафти в ускладнених умовах: Монографія. Івано-Франківськ: Нова Зоря, 2013. 711 с.

3. Бомба А.Я., Гладка О.М., Кузьменко А.П. Обчислювальні технології на основі методів комплексного аналізу та сумарних зображень: [монографія]. Рівне: Ассоль, 2016. 283 с.

4. Бомба А.Я., Булавацький В.М., Скопецький В.В. Нелінійні математичні моделі процесів геогідродинаміки. К.: Наукова думка, 2007. 308 с.

5. Бомба А.Я., Каштан С.С., Пригорницький Д.О., Ярощак С.В. Методи комплексного аналізу: Монографія. Рівне: НУВГП, 2013. 415 с.

6. Bomba A.Ya., Yaroshchak S.V., Myslyuk M.A. (2013). Mathematic modelling of thermodynamic effects in a gas formation well bore zone, *Journal of Hydrocarbon Power Engineering*, №1, 1–4.

7. Бомба А.Я. Кузьменко А.П., Гладка О.М. Синтез числових методів конформних відображень та сумарних зображень при моделюванні ідеальних полів для криволінійних областей. *Вісник Київського нац. ун-ту ім. Т. Шевченка. Серія: фіз.-мат. науки*. 2012. № 2. 87–94.

8. Bomba A.Ya., Hladka O.M. (2017). Problems of identification of the parameters of quasiideal filtration processes in nonlinear layered porous media, *Journal of Mathematical Sciences*, № 2 (220), 213–225.

9. Bomba Andrey Ya., Hladka Elena N. (2014). Methods of Complex Analysis of Parameters Identification of Quasiideal Processes in Nonlinear Doubly-layered Porous Pools**,** *Journal of Automation and Information Sciences,* Vol.46, №11, 50–62.

10. Hladka O., Bomba A. (2014). The complex analysis method of numerical identification of parameters of quasiideals processes in doubly-connected nonlinear-layered curvilinear domains, *J. of Math. and System Sci.,* Vol. 4, № 7, 514–521.

11. Ляшко И.И., Великоиваненко И.М. Численно-аналитическое решение краевых задач теории фильтрации. К.: Наукова думка, 1973. 264 с.

12. Polozhii G. N. (1965). The method of summary representations for numerical solution of problems of mathematical physics. Pergamon Press, London.
13. Гладка О.М. Системний підхід до математичного моделювання фільтраційних процесів у багатозв'язних криволінійних LEF-пластах. *Системні дослідження та інформаційні технології*, 2016. № 2. 58–73.
14. Василевский Ю.В., Ольшанский М.А. Краткий курс по многосеточным методам и методам декомпозиции области. М: МГУ, 2007. 105 с.
15. Тихонов А.Н., Арсенин В.Я. Методы решения некорректных задач. М.: Наука, 1986. 288 с.
16. Vogel C.R. (2002) Computational methods for inverse problems. Philadelphia: SIAM. 183 p.
17. Kabanikhin S.I. (1995) Numerical analysis of inverse problems. *J. of Inverse Ill-posed Problems*. 3, No. 4. 278–304.
18. Сергиенко И.В., Дейнека В.С. Идентификация параметров системы конвективно-диффузионного переноса. *Кибернетика и системный анализ*, 2009. № 1. 42–63.
19. Pomp A. (1998) The boundary-domain integral method for elliptic system. With application in shells. *Lecture Notes in Mathematics*. Vol. 1683. Berlin, Heidelberg: Springer-Verlag. 163 p.

Information about authors:

Hladka O. M.,

PhD on Engineering Science, Associate Professor

Karpovych I. M.,

PhD on Physics and Mathematics Science, Associate Professor

Department of Computer Science,

National University of Water and Environmental Engineering

11 Soborna St., Rivne, Ukraine